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Signal Extraction in ARIMA Time Series Program SEATS

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Signal Extraction in Arima Time Series

PROGRAM SEATS

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Abstract

This document explains a program for estimation of unobserved components (or signals) in univariate time series. The program follows the ARIMA-model-based method first developed by Burman (1980) and by Hillmer and Tiao (1982); it originated, in fact, from the seasonal adjustment program developed by Burman at the Bank of England.

The program fits, first, an ARIMA model to the series, and provides a detailed diagnosis. It identifies, next, the components present in the series; these are typically the trend, seasonal, and irregular components, although a separate cyclical component can also be estimated. The ARIMA models for the components are fully specified. Minimum mean square error (MMSE) estimates of the components are computed, as well as their forecasts. For each component, standard errors are provided for the different type of estimators (concurrent, preliminary, and historical or final estimator) and forecasts.

The structures of the theoretical component and of its MMSE estimator are analysed, and compared to the estimate actually obtained. This comparison yields additional diagnostic elements. The last part of the program contains information of applied interest, concerning the properties of the different signals used in practice (mostly, the seasonally adjusted series and the trend). A detailed analysis is made of the different types of error (i.e., the magnitude of the revisions the estimators will undergo, the duration of the revision period, the error in the final estimator), and how they affect the behavior of the rates-of-growth measures used in short-term monitoring and policy making.

Preliminary Version: December 1991

PROGRAM SEATS
(Signal Extraction in Arima Time Series)

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1. What the Program Does: Overview

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The program falls into the class of so-called ARIMA-model-based methods for decomposing a time series into its unobserved components (i.e., for extracting from a time series its different signals). The method was originally devised for seasonal adjustment of economic time series (i.e., removal of the seasonal signal), and the basic references are Burman (1980), and Hillmer and Tiao (1982). Both methods are closely related to each other, and to the one followed in this program. In fact, SEATS developed from a program built by Burman for seasonal adjustment at the Bank of England (1982 version) and its present version still contains parts of Burman's program; for this, and for many fruitful comments and discussions throughout these years, I wish to express to him my deepest gratitude. Thanks are also due to Gabriele Fiorentini and Gianluca Caporello; without their help, this (preliminary) program would have never been completed. The program, in its present version, is being used at the Bank of Spain for seasonal adjustment of economic series.

The program starts by fitting an ARIMA model to the series. Let X_t denote the original series, x_t its Box-Cox type of transformation (most often, $x_t = \log X_t$), and

$$z_t = D(B)\, x_t \,, \tag{1}$$

represent the "differenced" series, where B stands for the lag operator, and D(B) denotes the differences taken on x_t in order to (presumably) achieve stationarity. In SEATS,

$(1-B)^d (1-B^{MQ})^{BD}$

$$D(B) = \nabla^D \nabla_{MQ}^{BD} \,, \tag{2}$$

where $\nabla = 1 - B$, and $\nabla_{MQ}^{BD} = (1 - B^{MQ})^{BD}$ represents seasonal differencing of period MQ. (See Section 3.)

The model can be expressed as

$$\phi(B)\, z_t = \theta(B)\, a_t + \mu \,, \tag{3}$$

where

- μ is a constant,

- a_t is a white-noise series of innovations, normally distributed with zero mean and variance σ_a^2 .

$\phi(B)$ and $\theta(B)$ are autoregressive and moving average polynomials in B, respectively, which can be expressed in multiplicative form as the product of a regular polynomial in B and a seasonal polynomial in B^{MQ}, say:

$$\phi(B) = \phi_r(B)\ \phi_s\left(B^{MQ}\right), \tag{4a}$$

$$\theta(B) = \theta_r(B)\ \theta_s\left(B^{MQ}\right). \tag{4b}$$

Putting together (1) - (4), the complete model can be written in detailed form as

$$\phi_r(B)\ \phi_s\left(B^{MQ}\right)\ \nabla^D\ \nabla_{MQ}^{BD}\ x_t = \theta_r(B)\ \theta_s\left(B^{MQ}\right) a_t + \mu, \tag{5a}$$

and, in concise form, as

$$\phi(B)\ D(B)\ x_t = \theta(B)\ a_t + \mu, \tag{5b}$$

or

$$\Phi(B)\ x_t = \theta(B)\ a_t + \mu, \tag{5c}$$

where $\Phi(B) = \phi(B)\ D(B)$ represents the complete autoregressive polynomial, including all unit roots.

Notice that, if P denotes the order of $\phi(B)$, Q the order of $\theta(B)$, and DT the order of $D(B)$, so that $DT = D + BD \cdot MQ$, then the order of $\Phi(B)$ is $PT = P + DT$. The program SEATS requires that $PT \geq Q$; in practice, this constraint is of little importance.

It should be pointed out that the autoregressive polynomial $\phi(B)$ is allowed to have unit roots, which are estimated typically with considerable precision. For example, unit roots in $\phi(B)$ would be present if the series were to contain a nonstationary cyclical component (as in Example 2), or if the series had been underdifferenced. They can also appear as nonstationary seasonal harmonics.

The program decomposes a series that follows model (5) into several components. The decomposition can be multiplicative or additive. Since the former becomes the second by simply taking logs, we shall use in the discussion an additive model, such as

$$x_t = \sum_i x_{it}, \tag{6}$$

where x_{it} represents a component. The components that SEATS considers are:

x_{pt}	=	the TREND component,
x_{st}	=	the SEASONAL component,
x_{ct}	=	the CYCLICAL component,
x_{ut}	=	the IRREGULAR component.

The trend component represents the long-term evolution of the series and displays a spectral peak at frequency 0. The seasonal component captures the spectral peaks at seasonal frequencies, and the cyclical component captures periodic fluctuation with period longer than a year, and will have a spectral peak for the associated frequency, between 0 and $(2\pi / MQ)$. Finally, the irregular component captures erratic, white-noise behavior, and hence has a flat spectrum. The components are determined and fully derived from the structure of the (aggregate) ARIMA model for the observed series, model (5), which can be directly identified from the data. The program is mostly aimed at monthly or lower frequency data. It is not often that monthly, or even quarterly, ARIMA models identified for macroeconomic time series exhibit a cyclical component with constant period and frequency.

The decomposition assumes orthogonal components, and each will have in turn an ARIMA expression. In order to identify the components, we will require (except for the irregular one) that they be clean of noise. This is called the "canonical" property, and implies that no additive white noise can be extracted from a component that is not the irregular one. The variance of the latter is, in this way, maximized, and, on the contrary, the trend, seasonal and cycle are as stable as possible (compatible with the stochastic nature of model (5)). Although an arbitrary assumption, since any other admissible component can be expressed as the canonical one plus independent white-noise, it seems sensible to avoid contamination of the component by noise, unless there are a-priori reasons to do so.

In brief, the program does the following (at present, however, the output is ordered in a different way).

First, the ARIMA model is estimated. The model is analyzed and a detailed diagnosis is performed. Next, a frequency domain partition of the model into its components provides the filters that yield the series components estimates. The estimators are obtained as the conditional expectations given the observed series, and are thus minimum MSE estimators. Forecasts of the series and of all components are also computed. Standard errors are obtained for the different types of estimators (concurrent, preliminary, and historical or final estimator) and for the forecasts.

Besides the estimation part of the components and forecasts, the program performs a detailed analysis of the decomposition. The ARIMA models for the unobserved components are derived and analyzed. The structure of the theoretical minimum MSE estimator is obtained and compared with the one obtained empirically; this provides additional elements of diagnosis. The last part of the program presents information of applied interest, concerning the properties of the different signals used in practice (mostly, the seasonally adjusted series and the trend). The different types of estimation errors are analysed (i.e., the size of the revisions the estimator will undergo, the duration of the revision period, the error in the final estimator, etc.), and it is seen how these errors affect the behavior of the rates-of-growth measures used in short-term monitoring and policy making.

PROGRAM SEATS
(Signal Extraction in Arima Time Series)

2. Instructions for Installation and Execution

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INSTALLATION

Insert the diskette in drive A or B, and
change the default drive (type "A:" or "B:").
When the prompt appears type:

INSTALL

The installation procedure creates a directory "SEATS";
be sure it doesn't already exist.

If you have a partitioned diskette, you will be asked in which drive the program should be written (from C to G).

TO RUN THE PROGRAM

Once in **SEATS**, type:

INPUT

This is a very simple usable program to prepare the input file for the main program (ESTIM). It shows a list of all files in the directory SERIES (up to a maximum of 75). If the user series is not in the directory SERIES, type "88" until the message

"PLEASE TYPE THE NAME OF YOUR SERIES"

appears. Then type the filename, with the "path" extension if needed, and follow the normal procedure to set the parameters.

Once you have selected a series, it is possible to set the values of the program parameters (showing a list of all default values).

If the option "77" (NOSERIES) is used, no series is entered, and SEATS runs for an ARIMA model. The program sets the parameter NOSERIE = 1, and then INIT = 2 (no estimation), and LAM = 1. (The program also sets the variance of the model innovations equal to 1.) The rest of the parameters are entered in the usual manner. In this case, the user finds the output in the file "NOSERIE.OUT".

The program allows some simple facilities:

"q"	to exit
"h"	to see default parameters
"l"	to see already set parameters.

Set the parameters typing them (one by one) according to the following simple syntax:

Parameter-name = *Parameter-value*

Once all (non-default) parameter values have been set, in the next line type "end". This creates the file "series", containing the data and parameter values ready for the main program.

(Of course, the file "series" can be edited directly.)

The file of your own series must be written according to the following format (see also the example):

****	first line
seriesname	second line
number of observations (NZ)	third line
starting year	third line
starting period	third line
frequence (num of obs./year)	third line
observations (in any format)	fourth to nth line
****	n + 1 line

The year, starting period, and frequence are used for the format of the tables.

<u>Warning</u>:
The series cannot contain more than 250 observations (NZ ≤ 250).

Next, type:

ESTIM

This is the main program. After you have executed the INPUT, type "ESTIM" to run the program.

OUTPUT FILE

You can see the result of the program by editing or printing the file *seriesname* OUT.

GRAPHICS

Typing "GRAPH" you can see some graphics on the screen. The program selects the better graphic resolution for you (if you have any graphic adapter). In this version it also supports the print screen for a VGA graphic adapter with monochrome video.

Note: To use the print screen, type the DOS command "GRAPHICS' from DOS prompt.

The program "GRAPH" creates the subdirectory GRAPH in SEATS. All relevant arrays produced by the output of the program ESTIM are passed to this subdirectory, from which they can be retrieved for further use in programs such as SAS, MATLAB, GAUSS, or other econometrics/statistics/graphics package. This use can be for further numerical analysis, or for different editing of the graphs, which can then be sent to a laser printer.

LASER PRINT (only for HP LASER JET):

Direct laser jet prints of the graphics produced by SEATS can be obtained in the following way:

1) From the prompt, type EGALASER.
ex. C: > EGALASER

2) Run the program GRAPH with the oprion "-P".
ex. C:> GRAPH -P

3) Graph creates a file "GRAPH.LHJ"; to print it, type:
COPY /B GRAPH.LHJ LPT1

Notice that several graphs can be jointly combined through the OVERLAY facility.

EXAMPLE OF INPUT FILE

****

SIMUL 1

80	1931 1 4		
908.889	900.955	898.398	930.681
929.432	916.135	900.841	921.783
914.894	906.257	903.142	935.797
932.514	918.795	903.560	925.507
919.861	912.476	908.766	941.180
939.078	925.599	909.574	930.882
925.436	920.198	916.806	948.492
944.450	930.076	914.459	936.719
932.627	927.826	925.762	956.722
950.648	935.269	920.326	944.248
938.790	935.342	934.148	966.003
958.164	942.723	925.738	950.206
947.084	944.789	942.702	973.542
965.161	948.695	931.660	957.617
955.769	955.464	950.859	978.271
969.013	955.175	940.039	965.500
965.167	964.822	958.881	984.952
977.121	965.469	950.574	975.042
973.951	973.724	967.949	992.984
983.713	970.250	956.349	982.276

****

Note:
The first and last lines with the four stars, as well as the second line with the series name, should all start on the first column.

PROGRAM SEATS
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3. Description of the Model and of the Input Parameters

ARIMA MODELS

p d q P D Q

Multiplicative model: $(P,\ D,\ Q) \times (BP,\ BD,\ BQ)_{MQ}$

where $\quad P \le 3,\quad D \le 3,\quad Q \le 3$

$\quad BP \le 1,\quad BD \le 2,\quad BQ \le 1\,.$

(In principle, some of the numbers can be increased, but I would not recommend doing it at present.)

(Restriction: $P + D + BP \cdot MQ + BD \cdot MQ \ \ge\ Q + BQ \cdot MQ$.)

<u>Model</u>:

$$\phi_P(B)\ \Phi_{BP}\left(B^{MQ}\right)\ \nabla^d\ \nabla_{MQ}^{BD}\ x_t \ =\ \theta_Q(B)\ \Theta_{BQ}\left(B^{MQ}\right) a_t + \mu\ ,$$

where, for the most general case,

$$\nabla_{MQ}^{BD} \ =\ \left(1 - B^{MQ}\right)^{BD}$$

$$\begin{array}{ccccccc} \phi_P(B) = 1 & - & \phi_1 B & - & \phi_2 B^2 & - & \phi_3 B^3 \\ & & \Downarrow & & \Downarrow & & \Downarrow \\ & & \mathit{PHI(1)} & & \mathit{PHI(2)} & & \mathit{PHI(3)} \end{array}$$

$$\begin{array}{cc} \Phi\left(B^{MQ}\right) = 1 - & \Phi_1 B^{MQ} \\ & \Downarrow \\ & \mathit{BPHI(1)} \end{array}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3$$
$$\Downarrow \quad \Downarrow \quad \Downarrow$$
$$TH(1) \quad TH(2) \quad TH(3)$$

$$\Theta\left(B^{MQ}\right) = 1 - \Theta_1 B^{MQ}$$
$$\Downarrow$$
$$BTH(1)$$

MQ = *Number of observations per year (12 for monthly, 6 for bimonthly, 4 for quarterly, 1 for annual, and so on).*

IMEAN = 0 *No mean correction* $(\mu = 0)$.
= 1 *Mean correction* $(\mu \neq 0)$.

Other parameters: (D: default value)

TYPE = 0 *Maximum Likelihood Estimation.* (D)
= 1 *Constrained Least Squares.*

LAM = 0 *Logs are taken.* (D)
= 1 *Original Series.*

INIT = 0 *Starting Values for Param. computed automatically.* (D)
= 1 *Starting Values for Param. input. (*)*
= 2 *Starting Values for Param. input and no parameter estimation is done. (Parameters remain fixed; only residuals and forecasts are estimated.)*

(*) When INIT = 1 or 2, the inputed parameters are entered as PHI(i); TH(i); BPHI(i); BTH(i), as detailed in the previous page.

L	=	0 *ARIMA estimation only.*
	=	A positive integer *ARIMA estimation, plus the 1 to L periods ahead forecast.*
	=	-1 *ARIMA estimation, plus signal extraction, plus forecasting (determined by the program).* (D)
MAXIT	=	A positive integer. (D = 20) *Number of iterations in ARIMA estimation.*
EPSIV	=	A small positive number. (D = 10^{-3}) *Convergence criteria for estimation of ARIMA.*
M	=	A Positive Number. (D = 36) *Number of ACF and PACF printed in tables* $(M \leq 48)$.
IQ	=	A Positive Number. (D = 24) *Number of autocorrelations used in computing Q-statistics.*
NDEC	=	A Positive Number. (D = 2) *Controls the number of printed decimals in some of the tables.*
NDEC1	=	*As NDEC, but for another set of tables.* (D = 3)
SEK	=	A Positive Number. (D = 2.5) *Number of Standard Deviations used for the detection of outliers in residuals.*
HS	=	A Positive Number. (D = 10) *Sets the scale for the vertical axis of the spectral graphs.*
NPRINT	=	0 *No printing of estimation steps.* (D) 1 *Printing of estimation steps.*
FH	=	A positive integer. (D = 8) *Minimum number of forecast horizon for series and components when* L = -1.

EPSPHI = *When* $\phi_p(B)$ *contains a complex root, it is allocated to trend or seasonal if its frequency differs from the trend and seasonal frequencies by less than EPSPHI (measured in degrees).* (D = 5)

MATLAB = 1 *Creates a set of arrays for direct use in Matlab.*
0 (D = 0)

TA = 0 *Forces the residuals to have zero mean.*
A positive number: if the t-value of the mean of the residuals is larger than TA, the estimator of the constant in the model is modified. (D = 100, *meaning that no modification will ever be made!)*

XL and UR *When the modulus of an estimated root falls in the range (XL, 1), it is set equal to UR*
XL = .98 (D) ; UR = 1 (D)

NOSERIE = 0 *Usual case* (D)
= 1 *No series is used. An ARIMA model is entered* (INIT = 2, PHI (1) = _ , ...), *and the program performs the decomposition of the model and the subsequent model-based analysis.*

NPRFRA
IPASS1 = *At present, to be ignored.*
NANNUA

One only needs to enter the parameters that take values different from the default ones.

<u>Default</u> values for the model orders (and related)

P = 0 D = 1 Q = 1

BP = 0 BD = 1 BQ = 1

MQ = 12

IMEAN = 0

So, the default values estimate the model

$$\nabla \nabla_{12} \log x_t = (1 - \theta_1 B)(1 - \Theta_1 B^{12}) a_t \tag{1}$$

(the so-called Airline Model of Box and Jenkins, 1970), appropriate for monthly data, by a Maximum Likelihood method.

After ML estimation of model (1), the default option computes, then, 24 forecasts for the series, and estimates the trend, seasonal and irregular components in the series (together with 24 forecasts of the trend and seasonal). Standard errors for all the previous estimators and forecasts are also computed.

The program performs next a detailed analysis of the model, of the components and, in particular, of the trend and seasonally adjusted series, and of their rates of growth.

DEFAULT VALUES FOR THE PARAMETERS

TYPE = 0	INIT = 0
LAM = 0	IMEAN = 0
P = 0	D = 1
Q = 1	BP = 0
BD = 1	BQ = 1
MQ = 12	L = -1
SEK = 2.5D0	M = 36
IQ = 24	NDEC = 2
NDEC1 = 3	IPASS1 = 0
MAXIT = 20	EPSIV = 0.001D0
NPRFRA = 0	NANNUA = 1
HS = 10.0	FH = 8
TA = 100.0D0	EPSPHI = 5.0D0
NPRINT = 0	MATLAB = 0
NOSERIE = 0	XL = .98D0
UR = 1.0D0	

PROGRAM SEATS
(Signal Extraction in Arima Time Series)

4. Summary of the Program Output

a) ARIMA model estimation: results and diagnosis
b) Decomposition of the ARIMA model; Derivation of the models for the components
c) Distribution of the estimators and diagnosis
d) Error analysis
e) Estimates of the components

4. Summary of the Program Output

The output starts by displaying the tables of X_t , x_t , z_t and $z_t - \bar{z}$ if $\mu \neq 0$. The mean, variance, autocorrelation function (ACF), and partial autocorrelation function (PACF) for z_t , together with the associated standard errors, are next displayed.

a) ARIMA model estimation: results and diagnosis

The ARIMA estimation computes, first, initial values of the parameters. Maximum Likelihood estimation is carried out as in Osborn (1974), and conditional least squares as in Box and Jenkins (1970). When the modulus of an estimated AR root falls within the interval $1 \pm$ XL, where XL is a small number (.02 by default), it is fixed as 1, and the AR polynomial contains a unit root. MA roots are required to be invertible. (Estimation is done using transformed parameters, denoted X (PH1) , X (TH), etc. These transformed parameters are helpful in detecting when the nonstationary or noninvertible regions are reached.)

The estimates of the model parameters, their standard errors, and their correlation matrix are then displayed.

The residuals obtained in estimation are tabulated, and the ones larger in absolute value than SEK $\times \sigma_a$ listed and dated. Then come several statistics computed on the residuals:

- Mean (and t-test for whether it is zero),
- Skewness, Kurtosis, and their standard errors,
- Standard error, variance, and DW of the residuals,
- ACF, Ljung-Box Q statistics for residual autocorrelation, and Pierce Q_s statistics for residual seasonality,
- PACF,
- Asymptotic t-tests for randomness of residuals and of their ACFs (based on the distributions of runs),
- ACF of squared residuals and associated Q-statistics as a check of the linearity assumption.

Finally, the backward residuals, obtained with the model (5) with B replaced by $F = B^{-1}$ (the forward operator), are also printed. These residuals are needed to compute backcasts of the series, which in turn are needed to estimate the components for the first years of the sample.

b) Decomposition of the ARIMA model: Derivation of the models for the components

Since, having estimated the residuals a_t, it is always possible to divide them by their standard deviation, in this section I shall assume, without loss of generality, $\sigma_a^2 = 1$. Moreover, since μ can be removed from the model by centering z_t around its mean, we assume $\mu = 0$.

If the model for the ith component is given by

$$\phi_i(B)\, x_{it} = \theta_i(B)\, a_{it}\,, \tag{7}$$

where a_{it} is a n.i.i.d. $(0,\ \sigma_i^2)$ variable, then, from (6),

$$x_t = \sum_i \theta_i(B) / \phi_i(B)\, a_{it}\,, \tag{8}$$

where we assume that different components do not share the same AR root. (Since the AR roots represent spectral peaks, this assumption is harmless. The components differ precisely in that they capture peaks in the spectrum at different frequencies.) Removing the denominators from (8),

$$\left[\prod_i \phi_i(B)\right] x_t = \sum_i \tilde{\theta}_i(B)\, a_{it}\,, \tag{9}$$

where $\tilde{\theta}_i(B) = \theta_i(B)\, \tilde{\phi}_i(B)$, and $\tilde{\phi}_i(B)$ denotes the product of all components AR polynomials except $\phi_i(B)$; i.e.,

$$\tilde{\phi}_i(B) = \prod_{j(j \neq i)} \phi_j(B) = \Phi(B) / \phi_i(B)\,.$$

Comparing (9) with (5c), several results emerge:

1) The overall AR polynomial for x_t is equal to the product of all AR polynomials for the components, i.e.

$$\Phi(B) = \prod_i \phi_i(B)\,. \tag{10}$$

2) The right-hand side of (9) is the sum of uncorrelated finite moving averages, and hence it will accept an MA representation $\theta(B)\, a_t$, where

$$\theta\,(B)\,a_t \;=\; \sum_i \theta_i\,(B)\,\tilde{\phi}_i\,(B)\,a_{it}\,. \tag{11}$$

If $(P_i\,,\,Q_i)$ are the orders of the model (7) for component i, then

$$PT \;=\; \sum_i P_i\,,$$

and a reasonable assumption for the order of the components' MA polynomials is

$$Q_i \;=\; P_i\,.$$

In this case, it is easily seen that all the elements of the summation in the right-hand side of (11) are of the same order PT. The models for the components are, thus, "balanced" in the sense that the AR and MA parts have the same degree. The decomposition of the series will be such that

$$\frac{\theta\,(B)}{\Phi\,(B)}\,a_t \;=\; \sum_i \frac{\theta_i\,(B)}{\phi_i\,(B)}\,a_{it}\,. \tag{12}$$

Complessivo

Since $\Phi\,(B)$ is directly identified in the overall model, its roots will indicate the components present in the series. The roots of $\Phi\,(B) \,=\, 0$ are allocated as follows:

Roots of $\nabla^D \;=\; 0 \;\Rightarrow\; B = 1$ (D times) $\Rightarrow$ TREND

Roots of $\nabla^{BD}_{MQ} \;=\; 0 \;\Leftrightarrow$ BD times the roots of

$$(1\,-\,B)\,\underbrace{\left(1 \,+\, B \,+\, B^2 \,+\, \cdots \,+\, B^{MQ\,-\,1}\right)}_{\substack{\parallel \\ S\,(B)}} \;=\; 0\,,$$

where the root $B \;=\; 1 \;\Rightarrow$ TREND,

and the roots of $S\,(B) \;=\; 0 \;\Rightarrow$ SEASONAL.

Roots of $\phi_r\,(B) \;=\; 0 \;\Rightarrow$ real roots $>\,0 \;\Rightarrow$ TREND

real roots $<\,0 \;\Rightarrow$ imply a component of period 2, hence: SEASONAL, if frequency of data is larger than annual,

CYCLE, if annual data.

Complex roots $\Rightarrow$ SEASONAL, if frequency corresponds to seasonal frequency. (The interval for the seasonal frequency is controlled by EPSPHI.)

$\Rightarrow$ CYCLE, if frequency corresponds to a period longer than a year.

Roots of $\phi_s\left(B^{MQ}\right) = 0 \Leftrightarrow$ Roots of

$$(1 - \eta B)\left(1 + \eta B + \eta^2 B^2 + \cdots + (\eta B)^{MQ-1}\right) = 0 ,$$

where $\eta = (\mathrm{BPHI})^{1/MQ}$.

The real positive root $\Rightarrow$ TREND

The roots of $\left(1 + \eta B + \cdots + (\eta B)^{MQ-1}\right) = 0 \Rightarrow$ SEASONAL.

In the Second Part of the program printout, the polynomials in B of the overall model are listed, as well as the factorization of the AR one, and the allocation of the roots to the different components.

Having determined which components are present and their AR parts, the decomposition of the overall model is made in the frequency domain. The spectrum (or pseudospectrum) of the series is expressed as the ratio of harmonic functions. Thus, for example, in the printout:

$F(x)$ denotes the Fourier Transform of the ACF associated with $\theta(B)$, that is

$$\begin{aligned} F(x) &= \theta\left(e^{-i\omega}\right)\theta\left(e^{i\omega}\right) = \\ &= f_0 + f_1 \cos\omega + f_2 \cos 2\omega + f_3 \cos 3\omega + \cdots . \end{aligned}$$

The printout displays the coefficients f_0 , f_1 , f_2 , f_3 $\cdots$.

If the spectrum of $\theta(B) / \Phi(B)\, a_t$ is $F(x) / H(x)$, the program first obtains a decomposition of the type

$$\frac{F(x)}{H(x)} = \frac{UT(x)}{T(x)} + \frac{V(x)}{S(x)} + \frac{UC(x)}{C(x)} + QT\,, \tag{13}$$

where QT is a constant, and $T(x)$, $S(x)$, and $C(x)$ are the Fourier Transforms of the components' AR polynomials:

$\phi_i\left(e^{-i\omega}\right)\ \phi_i\left(e^{i\omega}\right)$,

expressed as harmonic functions (as linear expressions in $\cos(j\,\omega)$, $j = 0, 1, 2, \cdots$). This is done by partial fractions expansion of the ratios of harmonic functions in several steps

1) $$\frac{F(x)}{H(x)} = \frac{RT(x)}{H(x)} + QT\,,$$

2) $$\frac{RT(x)}{H(x)} = \frac{U(x)}{N(x)} + \frac{V(x)}{S(x)}\,, \qquad (H(x) = N(x)\ S(x)\,,)$$

3) $$\frac{U(x)}{N(x)} = \frac{UT(x)}{T(x)} + \frac{UC(x)}{C(x)}\,, \qquad (N(x) = T(x)\ C(x)\,.)$$

(The last step is only carried out, of course, if there is a cycle component.)

Having obtained the decomposition (13), the additive components of the r.h.s. represent a first decomposition of the series. If $g(\cdot)$ denotes the spectrum, this first decomposition provides:

$g_p^0(x) = \dfrac{UT(x)}{T(x)}$ Spectrum of Trend

$g_s^0(x) = \dfrac{V(x)}{S(x)}$ Spectrum of Seasonal

$g_c^0(x) = \dfrac{UC(x)}{C(x)}$ Spectrum of Cycle

$g_u^0(x) = QT$ Spectrum of Irregular.

If there are unit AR roots the word spectrum is also used to refer to the "pseudo-spectrum".

To obtain the final decomposition, given by the balanced canonical components, plus white-noise irregular (with maximum variance), the program computes:

$$g_p(x) = g_p^0(x) - \min g_p^0(x)$$

$$g_s(x) = g_s^0(x) - \min g_s^0(x)$$

$$g_c(x) = g_c^0(x) - \min g_c^0(x)$$

$$g_u(x) = g_u^0(x) - \min g_u^0(x)\,.$$

When no decomposition with all spectra nonnegative can be obtained, the program informs that the attempted decomposition is invalid.

For more details on the decomposition achieved, additional helpful papers are Bell and Hillmer (1984), Maravall and Pierce (1987), and Maravall (1989).

With these functions, the program proceeds to compute minimum MSE estimators of the components (the irregular estimate is obtained as a residual). The estimators are obtained by applying the so-called Wiener-Kolmogorov (WK) filter, using the algorithm suggested by Wilson (see Burman, 1980). The validity of the WK filter for nonstationary series can be found in Bell (1984) or Maravall (1988).

The estimates of the components, however, are presented at the end of the printout, in part 4. Before, the program provides a substantial amount of additional information. The first part of this information is mostly of analytical interest; the second part is oriented towards its application.

Once the spectra of the components are known, the Inverse Fourier Transform provides their ACFs. Factorization of the ACFs yields the ARIMA models for the components.

Using as an illustration Example 2 enclosed, the overall estimated ARIMA model is given by

$$(1 - 1.395\,B + B^2)\,\nabla\,\nabla_4\,x_t = (1 - 1.008\,B + .268\,B^2)\,a_t\,,$$

with $\sigma_a^2 = 1.022$.

The factorization of the AR polynomial yields:

$\nabla \nabla_4 \Rightarrow \nabla^2 = 0 \Rightarrow$ TREND.

$1 + B + B^2 + B^3 = 0 \Rightarrow$ SEASONAL.
Notice that $(1 + B + B^2 + B^3) = (1 + B^2)(1 + B)$, where $1 + B^2 = 0$ represents the once-a-year seasonal frequency (with a peak at $\omega = \pi/2$), and $1 + B = 0$ the twice-a-year seasonal frequency (with a peak at $\omega = \pi$).

$1 - 1.395 B + B^2 = 0 \Rightarrow$ CYCLE, with period = 7.9 quarters, and modulus = 1 (and hence nonstationary).

The models for the components are found to be:

Trend Component:

$$\nabla^2 p_t = (1 - .299 B + .701 B^2) a_{pt} ,$$

Seasonal Component:

$$(1 + B + B^2 + B^3) s_t = (1 - 1.538 B - 1.222 B^2 - .162 B^3) a_{st} ,$$

Cycle Component:

$$(1 - 1.395 B + B^2) c_t = (1 - .654 B + .364 B^2) a_{ct} ,$$

and u_t a white-noise variable. The variances of the components' innovations are

$$\sigma_p^2 = .032, \quad \sigma_s^2 = .103, \quad \sigma_c = .056, \quad \sigma_u = .025 .$$

The program also computes the model for the seasonally adjusted (S.A.) series, $x_t^a = x_t - s_t$, equal in this case to

$$(1 - 1.395 B + B^2) \nabla^2 x_t^a = (1 + 1.659 B - .905 B^2 + .091 B^3 + .046 B^4) a_{nt} ,$$

with $\sigma_n^2 = .359$. (Since $\sigma_a^2 = 1$, all component variances are expressed as a fraction of σ_a^2 .)

The roots of the MA parts of the components are also printed. The canonical property implies that their spectra have to present a zero, which, in turn, implies the presence of a unit MA root. For the trend and cycle, as is always the case, the zero happens for the highest frequency $\omega = \pi$; for the seasonal, it falls in the interval $(\pi/2, \pi)$.

c) Distribution of the estimators and diagnosis

Once the models for the components are known, it is possible to expand the analysis further, so as to provide elements for diagnosis and for inference.

In compact notation, if the overall model for x_t is

$$\Phi (B) x_t = \theta (B) a_t , \tag{14}$$

and, for component x_{it} :

$$\phi_i (B) x_{it} = \theta_i (B) a_{it} ,$$

the optimal estimator obtained with the WK filter can be expressed as

$$\hat{x}_{it} = \sigma_i^2 \frac{\theta_i (B) \ \theta_i (F)}{\phi_i (B) \ \phi_i (F)} \frac{\Phi (B) \ \Phi (F)}{\theta (B) \ \theta (F)} x_t ,$$

or, using (14),

$$\phi_i (B) \hat{x}_{it} = \theta_i (B) \ \beta (F) \ a_t , \tag{15}$$

where $\beta (F)$ is the convergent expression in F:

$$\beta (F) = \sigma_i^2 \frac{\theta_i (F) \ \tilde{\phi}_i (F)}{\theta (F)} ,$$

where $\tilde{\phi}_i (F)$ has already been defined. Therefore, comparing (14) and (15) it is evident that, despite the similarities (for example, the stationarity inducing transformations are the same) the models for the theoretical component and for the theoretical minimum MSE estimator are not the same, and in particular, their stationary transformation will have different ACFs. Therefore, once the component estimate has been obtained, it should not be compared to the theoretical component, but to the theoretical optimal estimator. This comparison can provide the basis for a proper diagnostic of the results, to assess whether a given decomposition, based on an aggregate ARIMA model, is in agreement with the data. A more complete discussion of this point can be found in Maravall (1987). Of course, a similar reasoning applies to the Crosscovariance Functions (CCF): although the theoretical components are uncorrelated, their optimal estimators will be correlated. From (15), the CCFs between any two estimators are easy to obtain.

The program compares the ACFs and CCFs of the theoretical components, the theoretical estimators, and the estimates actually obtained. Boadly, the latter two should be similar, and in particular their variances should always underestimate the variance of the theoretical component.

d) Error analysis

Next, the program analyses the different types of estimation error. This part is explained in some detail in the enclosed background papers. In brief, let $\hat{x}_{it}$ be the optimal estimator that the WK filter would provide for a series going from $-\infty$ to ∞. In practice, only a finite series is available, say x_1, x_2, $\cdots$, x_T. Denote the estimator of x_{it} obtained with the finite series as $\hat{x}_{it}(T)$. (To simplify the discussion, we assume that the first observation x_1 is "far" from period t.) This estimator is a preliminary one, and its preliminary character will vary with different values of T. As T increases, the preliminary estimator will be subject to revisions, and will converge towards the final estimator $\hat{x}_{it}$.

Let δ_{it} denote the error in the final estimator:

$$\delta_{it} = x_{it} - \hat{x}_{it},$$

and $r_{it}(T)$ denote the revision error contained in the preliminary estimator $\hat{x}_{it}(T)$, that is:

$$r_{it}(T) = \hat{x}_{it} - \hat{x}_{it}(T).$$

Obviously, the total error in $\hat{x}_{it}(T)$ is the sum of the two:

$$e_{it}(T) = x_{it} - \hat{x}_{it}(T) = \delta_{it} + r_{it}(T);$$

moreover, the two estimation errors, δ_{it} and $r_{it}(T)$, are orthogonal to each other.

The output displays the variances and ACFs of the final estimation error, δ_{it}, of the revision error in the concurrent estimator, $r_{iT}(T)$, and of the total estimation error in the concurrent estimator, $e_{iT}(T)$. (The ACFs are needed to compute later the errors in the different rates of growth of the components estimates.) This is done for the trend, the S.A. series, and the cycle.

For illustration, let us consider Example 1 enclosed, where the program is run in its default option on the series of the Spanish monetary aggregate. The default ARIMA model fit is the Airline model, which is decomposed into trend, seasonal, and irregular components. The default model provides a roughly decent fit, although the ACF of the squared residuals points towards the presence of some, mostly seasonal, nonlinearity.

Concerning the components estimation errors, several results can be noticed:

(a) The variances of the final estimation error and of the revision in the concurrent estimator, for both the trend and S.A. series, are of roughly the same order of magnitude (close to 1/5 of the one-period-ahead forecast error variance for the series).

(b) The revision error is slightly larger than the final estimation error, although the differences are very small.

(c) The variance in the errors in estimating the trend is, in all cases, close to 15% larger than that for the S.A. series.

In summary, the variance of the error in the concurrent estimator of the trend is, approximately, 43% of σ_a^2; for the S.A. series this proportion is 36%.

In the Airline Model

$$\nabla \nabla_{12} x_t = \left(1 - \theta_1 B\right)\left(1 - \theta_{12} B^{12}\right) a_t + \mu ,$$

the parameters θ_1 and θ_{12} are associated with the stability of the trend and seasonal component, respectively. Since, in the limit, when they are equal to one, the component becomes deterministic, the closer θ_1 is to 1, the more stable the trend will be, and similarly, values of θ_{12} close to 1 are associated with a stable seasonal component. For the monetary aggregrate series $\theta_1 = -.262$, $\theta_{12} = .684$, and hence the series contains a rather unstable trend and a more stable seasonal. Furthermore, in this example, the derived models for the components show that the innovation variance for the trend is .28, while that for the seasonal is .04. It should be no surprise, thus, that the trend component is estimated with a larger error.

The program next indicates how fast the concurrent estimator converges towards the final estimator. Thus, in terms of the standard error, after one year, 42% of the revision in the trend and 31% of the revision in the S.A. series has been completed. After five years, these percentages become 87 and 85%, respectively. Although measured concurrently with more error, the trend estimator tends to converge faster.

Since seasonal adjustment of economic time series is typically done once a year, the factors used in practice to adjust for seasonality in the months between adjustments are forecasted factors. The next table indicates the decrease in precision of the factors used as we move away from the last adjustment. The program informs us that the move from the once-a-year adjustment to a concurrent one (i.e., to adjusting every month) would reduce (on average) the error in the factors used by 12%.

The output displays then, in detail, the recent estimates (for the last 24 months) of the components, as well as of their standard errors, indicating which part is associated with the revision. The same is done for the 1- to 24-months-ahead forecasts of the series and of the components. It is seen that, overall, for the monetary aggregate series, the forecasts of the series, the trend, and the S.A. series have associated standard errors of similar magnitude; overall, the trend forecast tends to be slightly more precise.

As for the precision of the seasonal factor, the output displays the 95% and 70% confidence intervals, for the final and the concurrent estimators.

Since monetary policy is typically stated in terms of the rate of growth, instead of on the direct monthly observation, the program computes next the errors in the estimators of different rates of growth of the components. The rates are all expressed in points of annualized percent growth, and the errors have been computed using linear approximations.

The first rate considered is the one most often used: the monthly rate of growth of the monthly series. Since this rate is too erratic, the program also looks at the monthly rate of a three-month moving average (centered, that is, using the 1-month-ahead forecast).

The next rate included in the program is particularly appropriate for series for which there is an (explicit or implicit) annual target. The output exhibits the precision of the measurement of the annual rate of growth as the year is being completed. Thus, for example, in August, the standard error of the rate of growth for the year is close to 1/3 of the standard error for the January measurement.

If the previous rate has a fixed horizon and spans a variable number of months, the last rate considered by the program has a fixed length of 12 months, and a variable horizon. It measures the annual growth by comparing the 6-months-ahead forecast with the observation one year before this horizon. It provides a measure of the "instantaneous" annual rate of growth. It is seen that, for the monetary aggregate series, this instantaneous computation of the annual growth is a more stable measure than the annualized monthly rates of growth. It is further seen that the centered estimator of the present rate of annual growth is better measured over the trend than over the S.A. series, which in turn provides a more precise measure than the one based on the raw series.

The output displays the standard errors of the previous rates of growth estimated concurrently, of their successive revised estimators, and of the final one. For example, 95% confidence intervals around the annualized rate of growth of the monthly S.A. series (or the trend) implied by the March observation, would be given, broadly, by:

± 4 percent points (p.p.) if measured over the monthly series;

± 3.3 p.p. if measured over the three-month moving average.

Moreover, the measurement of the rate of growth for the actual year (that uses the first three months already observed) has a 95% confidence interval of ± 2.4 p.p. Finally, a similar interval for the measurement of the centered estimator of the present rate of annual growth is 2 p.p.

e) <u>Estimates of the components</u>

Having finished the part on error analysis, the output ends with the tables displaying the complete set of estimates of the components (or factors if the decomposition is multiplicative) for the original series X_t.

PROGRAM SEATS
(Signal Extraction in Arima Time Series)

5. Example 1: SEASONAL ADJUSTMENT OF THE SPANISH MONETARY AGGREGATE (Default Option of the Program)

The example follows roughly the discussion in the enclosed paper "The Use of ARIMA Models in Unobserved-Components Estimation: An Application to Spanish Monetary Control". While the paper used the period 1973-1985, the printout covers the 234 months January 1972-June 1991.

The program estimates the model

$$\nabla \nabla_{12} \log x_t = \left(1 - \theta_1 B\right)\left(1 - \theta_{12} B^{12}\right) a_t + \mu \qquad (1)$$

to the Spanish monetary aggregate series. The series is decomposed into trend, seasonal and irregular components.

In this example, the input parameters are all set at their default value, except for IMEAN=1, corresponding to the presence of μ . Model (1), often referred to in short as the "Airline Model", was discussed in some detail by Box and Jenkins (1970). It has been our experience that it provides a convenient and useful starting point for many monthly macro-economic time series exhibiting trend and seasonality.

The series of the Spanish monetary aggregate (ALP) is used merely as an example. The analysis and results do not reflect official seasonal adjustment of the monetary aggregate by the Bank of Spain. Besides, prior to running SEATS on the series, three strongly negative additive outliers were modified with intervention analysis (using one single dummy variable for the three outliers).

SIGNAL EXTRACTION IN 'ARIMA' TIME SERIES

BY AGUSTIN MARAVALL & VICTOR GOMEZ, with the assistance of GABRIELE FIORENTINI and GIANLUCA CAPORELLO (VERSION 1991)

(Based on an original program developed by J.P. BURMAN at the Bank of England, version 1982)

FIRST PART:
ARIMA ESTIMATION

SERIES TITLE: alp234in

METHOD - MAXIMUM LIKELIHOOD

NO OF OBSERVATIONS =234

ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	.245	.246	.250	.254	.258	.262	.269	.273	.275	.280	.284	.293
1973	.302	.304	.310	.316	.321	.329	.339	.344	.348	.354	.359	.369
1974	.378	.378	.384	.391	.396	.402	.412	.414	.416	.423	.428	.440
1975	.451	.450	.456	.462	.467	.475	.489	.496	.499	.505	.511	.526
1976	.538	.537	.542	.549	.556	.564	.581	.586	.589	.598	.606	.623
1977	.639	.639	.646	.657	.664	.675	.695	.704	.709	.716	.720	.742
1978	.756	.754	.764	.779	.791	.805	.834	.840	.848	.858	.864	.890
1979	.912	.914	.928	.944	.951	.966	.998	.999	1.002	1.015	1.023	1.060
1980	1.079	1.075	1.089	1.110	1.114	1.129	1.163	1.177	1.184	1.198	1.205	1.235
1981	1.253	1.250	1.265	1.285	1.298	1.309	1.349	1.367	1.374	1.392	1.400	1.441
1982	1.468	1.468	1.490	1.521	1.541	1.559	1.604	1.620	1.634	1.658	1.664	1.697
1983	1.736	1.734	1.756	1.786	1.802	1.811	1.861	1.873	1.888	1.906	1.910	1.966
1984	2.000	1.990	2.010	2.031	2.040	2.068	2.143	2.151	2.168	2.191	2.185	2.249
1985	2.274	2.283	2.311	2.344	2.364	2.395	2.443	2.442	2.463	2.482	2.485	2.547
1986	2.574	2.583	2.619	2.643	2.655	2.696	2.744	2.731	2.752	2.772	2.775	2.855
1987	2.887	2.889	2.926	2.966	2.988	3.013	3.075	3.107	3.139	3.185	3.195	3.272
1988	3.313	3.305	3.349	3.393	3.410	3.429	3.493	3.505	3.532	3.570	3.580	3.670
1989	3.722	3.727	3.764	3.818	3.852	3.907	4.001	3.992	4.014	4.028	4.011	4.067
1990	4.078	4.057	4.099	4.165	4.199	4.261	4.344	4.348	4.368	4.389	4.401	4.527
1991	4.586	4.604	4.675	4.720	4.730	4.768						

TRANSFORMATION: Z -> LOG Z

TRANSFORMED SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	-1.406	-1.401	-1.387	-1.369	-1.353	-1.339	-1.313	-1.300	-1.290	-1.271	-1.258	-1.228
1973	-1.197	-1.190	-1.173	-1.151	-1.135	-1.113	-1.081	-1.066	-1.055	-1.037	-1.023	-.996
1974	-.974	-.973	-.958	-.939	-.927	-.912	-.887	-.881	-.876	-.861	-.849	-.822
1975	-.796	-.799	-.785	-.773	-.762	-.745	-.715	-.702	-.694	-.684	-.672	-.643
1976	-.620	-.623	-.612	-.599	-.587	-.572	-.544	-.534	-.529	-.514	-.502	-.473
1977	-.447	-.448	-.437	-.421	-.410	-.394	-.364	-.351	-.344	-.335	-.328	-.299
1978	-.280	-.282	-.269	-.250	-.235	-.217	-.182	-.174	-.165	-.153	-.146	-.117
1979	-.092	-.090	-.075	-.058	-.051	-.035	-.002	-.001	.002	.015	.023	.058
1980	.076	.072	.086	.104	.108	.121	.151	.163	.169	.181	.186	.211
1981	.225	.223	.235	.251	.261	.269	.300	.313	.318	.331	.337	.366
1982	.384	.384	.399	.419	.433	.444	.473	.482	.491	.505	.509	.529
1983	.551	.550	.563	.580	.589	.594	.621	.628	.636	.645	.647	.676
1984	.693	.688	.698	.709	.713	.726	.762	.766	.774	.784	.782	.810
1985	.822	.826	.838	.852	.861	.873	.893	.893	.902	.909	.910	.935
1986	.946	.949	.963	.972	.977	.992	1.009	1.005	1.012	1.020	1.021	1.049
1987	1.060	1.061	1.074	1.087	1.095	1.103	1.123	1.134	1.144	1.158	1.161	1.185
1988	1.198	1.196	1.209	1.222	1.227	1.232	1.251	1.254	1.262	1.273	1.275	1.300
1989	1.314	1.316	1.326	1.340	1.349	1.363	1.387	1.384	1.390	1.393	1.389	1.403
1990	1.405	1.401	1.411	1.427	1.435	1.450	1.469	1.470	1.474	1.479	1.482	1.510
1991	1.523	1.527	1.542	1.552	1.554	1.562						

NONSEASONAL DIFFERENCING D= 1
SEASONAL DIFFERENCING BD= 1

DIFFERENCED SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1973		.002	.003	.004	.000	.008	.005	.002	.001	-.001	.000	-.003
1974	-.007	-.006	-.002	-.003	-.004	-.007	-.006	-.009	-.006	-.003	-.002	.000
1975	.003	-.004	-.001	-.007	-.001	.002	.004	.008	.003	-.005	.001	.001
1976	-.003	.000	-.003	.000	.002	-.002	-.001	-.003	-.003	.005	.000	.000
1977	.003	.001	.000	.004	-.002	.001	.002	.002	.003	-.006	-.005	.000
1978	-.006	-.001	.002	.002	.004	.002	.005	-.005	.002	.002	.000	.001
1979	.005	.004	.002	-.001	-.009	-.002	-.003	-.006	-.006	.001	.001	.006
1980	-.006	-.006	-.002	.001	-.002	-.003	-.003	.010	.003	-.001	-.002	-.010
1981	-.004	.002	-.001	-.003	.005	-.005	.001	.002	-.001	.001	.000	.004
1982	.005	.002	.003	.005	.004	.003	-.002	-.004	.004	.001	-.002	-.009
1983	.004	-.001	-.002	-.003	-.005	-.006	-.002	-.003	-.001	-.005	-.001	.009
1984	-.005	-.004	-.003	-.006	-.005	.008	.008	-.003	.000	.001	-.005	.000
1985	-.006	.009	.003	.003	.005	-.001	-.016	-.004	.001	-.003	.004	-.004
1986	-.001	-.001	.002	-.005	-.004	.003	-.002	-.004	-.001	.000	.000	.003
1987	.001	-.003	-.001	.004	.003	-.007	.003	.015	.003	.007	.002	-.004
1988	.001	-.003	.000	.000	-.002	-.003	-.002	-.007	-.003	-.004	.000	.001
1989	.002	.004	-.003	.001	.004	.009	.005	-.006	-.002	-.007	-.007	-.011
1990	-.012	-.006	.000	.002	-.001	.001	-.005	.003	-.001	.001	.007	.014
1991	.011	.009	.005	-.006	-.006	-.007						

SERIES HAS BEEN MEAN CORRECTED

DIFFERENCED AND CENTERED SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1973		.003	.004	.004	.001	.009	.006	.003	.002	.000	.001	-.003
1974	-.007	-.006	-.002	-.003	-.004	-.007	-.006	-.009	-.006	-.002	-.002	.001
1975	.004	-.004	-.001	-.006	-.001	.003	.005	.008	.004	-.005	.001	.002
1976	-.002	.001	-.003	.001	.003	-.002	-.001	-.003	-.003	.006	.000	.001
1977	.003	.002	.001	.005	-.001	.002	.002	.003	.003	-.005	-.005	.000
1978	-.006	-.001	.003	.003	.005	.002	.006	-.004	.003	.002	.001	.001
1979	.006	.005	.002	-.001	-.008	-.002	-.002	-.006	-.006	.002	.001	.006
1980	-.006	-.006	-.001	.001	-.002	-.002	-.003	.011	.004	-.001	-.002	-.010
1981	-.004	.002	-.001	-.002	.006	-.004	.002	.002	-.001	.002	.000	.005
1982	.005	.002	.003	.005	.004	.003	-.001	-.003	.004	.002	-.002	-.009
1983	.004	.000	-.002	-.003	-.004	-.006	-.001	-.003	.000	-.005	-.001	.009
1984	-.005	-.003	-.003	-.006	-.004	.009	.009	-.002	.000	.002	-.005	.001
1985	-.006	.009	.003	.004	.005	-.001	-.015	-.004	.001	-.002	.004	-.004
1986	.000	.000	.002	-.004	-.004	.003	-.002	-.004	-.001	.000	.001	.004
1987	.001	-.002	-.001	.005	.003	-.006	.003	.016	.003	.007	.002	-.004
1988	.002	-.003	.001	.000	-.002	-.002	-.002	-.007	-.002	-.003	.000	.001
1989	.002	.004	-.003	.001	.004	.009	.006	-.005	-.002	-.007	-.007	-.010
1990	-.011	-.006	.001	.002	.000	.001	-.004	.003	.000	.001	.007	.015
1991	.011	.009	.006	-.006	-.006	-.006						

MEAN OF DIFFERENCED AND CENTERED SERIES .2904D-18

MEAN OF DIFFERENCED SERIES -.4345D-03

VARIANCE OF Z SERIES = .7487D+00

VARIANCE OF DIFFERENCED SERIES = .2080D-04

AUTOCORRELATIONS OF STATIONARY SERIES

	.3739	.1401	.0805	-.0542	-.0566	-.0385	-.0698	.0127	.0329	-.0428	-.1452	-.4101
SE	.0673	.0761	.0773	.0776	.0778	.0780	.0781	.0784	.0784	.0784	.0785	.0797
	-.2751	-.1068	-.0834	.0004	.0920	.0710	.0697	.0899	.0595	-.0512	.0090	-.0427
SE	.0888	.0925	.0931	.0934	.0934	.0939	.0941	.0943	.0947	.0949	.0950	.0950
	-.0205	-.0478	-.0810	-.1192	-.0352	-.0045	-.0263	-.0461	-.0978	.0271	.0103	-.0497
SE	.0951	.0951	.0952	.0955	.0962	.0963	.0963	.0963	.0964	.0968	.0969	.0969

PARTIAL AUTOCORRELATIONS

	.3739	.0004	.0326	-.1104	-.0050	-.0080	-.0471	.0627	.0112	-.0718	-.1467	-.3713
SE	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673
	.0041	.0506	-.0121	-.0080	.0482	-.0059	-.0203	.0875	.0777	-.1422	-.0197	-.2481
SE	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673
	-.0360	-.0733	-.0679	-.1159	.0813	.0394	-.0234	.0106	-.0260	.0244	-.0488	-.1969
SE	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673	.0673

```
MODEL FITTED

     NONSEASONAL     P= 0     D= 1     Q= 1
        SEASONAL    BP= 0    BD= 1    BQ= 1
     PERIODICITY    MQ= 12

INITIAL VALUES

  TH:    -.4494

 BTH:     .7087

 TRANSFORMED PARAMETERS:   -.4494    .7087

 MINIMUM BOUNDS         :   -.9800   -.9800
 MAXIMUM BOUNDS         :    .9800    .9800

 CONVERGED AFTER  4 ITERATIONS AND  13 FUTION VALUES    F = ,    .29613930E-02
       -.260217E+00         .685496E+00

PARAMETERS FIXED
    0


FINAL VALUES OF PARAMETERS FROM SEARCH


          X(THETA) = -.260217

            SE     =  .064078

          X(BTHETA)=  .685496

            SE     =  .050130
```

PARAMETER ESTIMATES

```
MEAN     = -.000434

SE       =  .000128
```

CORRELATION MATRIX

1.000	
.004	1.000

ARIMA PARAMETERS (B-J SIGNS)

```
THETA =    -.2602
SE    =     .0641

BTHETA=     .6855
SE    =     .0501
```

RESIDUALS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	.000	.001	-.002	.000	.001	-.004	-.002	.001	.000	.002	.000	.001
1973	.003	.003	.002	.003	.001	.006	.002	.002	.001	.001	.001	-.003
1974	-.004	-.002	.001	.000	-.003	-.002	-.003	-.006	-.003	-.001	-.001	-.001
1975	.001	-.006	.001	-.007	-.001	.001	.002	.003	.000	-.006	.002	.001
1976	-.002	-.003	-.003	-.003	.001	-.002	.001	-.001	-.002	.002	.000	.001
1977	.001	-.001	-.001	.002	-.001	.001	.002	.002	.001	-.005	-.003	.002
1978	-.005	.000	.001	.004	.003	.002	.007	-.004	.005	-.002	-.002	.002
1979	.002	.003	.003	.001	-.006	.002	.003	-.008	-.001	.002	-.001	.008
1980	-.006	-.002	.002	.002	-.006	.000	-.001	.006	.000	.000	-.002	-.004
1981	-.005	.002	-.001	.000	.002	-.006	.003	.005	-.001	.002	-.001	.002
1982	.000	.002	.003	.004	.004	-.002	.000	.001	.004	.002	-.003	-.007
1983	.006	.000	.000	.000	.000	-.006	.000	-.002	.003	-.004	-.001	.004
1984	-.003	-.002	-.002	-.005	-.003	.006	.006	-.005	.004	-.001	-.006	.005
1985	-.008	.010	-.001	.000	.002	.002	-.010	-.004	.004	-.003	.001	-.002
1986	-.005	.006	.001	-.005	-.001	.005	-.010	-.006	.003	-.002	.001	.003
1987	-.003	.002	.001	.002	.001	-.003	-.002	.011	.001	.006	.001	-.002
1988	.001	-.002	.002	.001	-.001	-.004	-.002	.001	.000	.001	.002	.000
1989	.002	.002	-.002	.003	.003	.005	.002	-.006	.000	-.006	-.004	-.009
1990	-.007	-.002	.000	.004	.001	.005	-.003	.001	-.001	-.002	.004	.007
1991	.003	.006	.004	-.004	-.003	-.002						

STUDENTIZED RESIDUAL OF 2.7550 AT T=158 (2 1985)

STUDENTIZED RESIDUAL OF -2.8546 AT T=163 (7 1985)

STUDENTIZED RESIDUAL OF -2.7805 AT T=175 (7 1986)

STUDENTIZED RESIDUAL OF 2.9031 AT T=188 (8 1987)

STUDENTIZED RESIDUAL OF -2.6182 AT T=216 (12 1989)

```
TEST-STATISTICS ON RESIDUALS
----------------------------

         MEAN=  -.6970D-04
      ST.DEV.=   .2285D-03
      OF MEAN
      T-VALUE=  -.3050

     SKEWNESS=  -.1974          ( SE =   .1601 )
     KURTOSIS=  3.3845          ( SE =   .3203 )

SUM OF SQUARES=  .2860D-02

 DURBIN-WATSON=  1.9949

STANDARD ERROR=  .3622D-02
OF RESID.
     VARIANCE=   .1312D-04
     OF RESID.
```

AUTOCORRELATIONS OF RESIDUAL

	.0014	.0403	.0997	-.1031	-.0183	.0336	-.0633	.0239	.1190	-.0714	-.0596	.0105
SE	.0654	.0654	.0655	.0661	.0668	.0668	.0669	.0672	.0672	.0681	.0684	.0686
	-.2270	-.0500	-.0783	-.0876	.1072	.0452	.0141	.0656	.0627	-.1011	.0293	-.0819
SE	.0686	.0718	.0719	.0723	.0727	.0734	.0735	.0735	.0738	.0740	.0746	.0747
	-.0291	-.0052	-.0738	-.0885	.0453	-.0406	-.0067	-.0079	-.1017	.0157	.0111	-.1223
SE	.0750	.0751	.0751	.0754	.0758	.0760	.0760	.0760	.0761	.0766	.0766	.0767

THE LJUNG-BOX Q VALUE IS 37.64 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (21)
THE PIERCE QS VALUE IS 1.70 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (2)

PARTIAL AUTOCORRELATIONS

	.0014	.0403	.0998	-.1058	-.0264	.0335	-.0419	.0153	.1157	-.0608	-.0870	-.0018
SE	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654
	-.1890	-.0518	-.0827	-.0385	.0829	.0354	.0302	.0326	.0743	-.0698	.0025	-.0963
SE	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654
	-.0141	-.1059	-.0985	-.1304	.0041	-.0054	.0405	.0123	-.0518	.0524	-.0111	-.1070
SE	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654	.0654

APPROXIMATE TEST OF RUNS ON AUTOCORRELATION FUNCTION

NUM.DATA= 36
NUM.(+)= 18
NUM.(-)= 18
T-VALUE= .33820

APPROXIMATE TEST OF RUNS ON RESIDUALS

NUM.DATA= 234
NUM.(+)= 117
NUM.(-)= 117
T-VALUE= .00000

AUTOCORRELATIONS OF SQUARED RESIDUAL

	.1271	.0477	-.0529	-.0472	.1321	.1529	.0709	.0420	-.0765	-.0608	-.0106	.2019
SE	.0654	.0664	.0666	.0667	.0669	.0680	.0694	.0698	.0699	.0702	.0704	.0705
	.1556	-.0141	-.0303	-.0253	.0797	.1379	.0493	.0138	-.1343	.0006	-.0450	.0582
SE	.0729	.0743	.0743	.0744	.0744	.0748	.0758	.0760	.0760	.0770	.0770	.0771
	.1638	-.0030	-.0599	.0210	-.0759	.0312	-.0054	-.0085	-.0087	-.0509	-.0402	-.0550
SE	.0773	.0788	.0788	.0790	.0790	.0793	.0793	.0793	.0793	.0793	.0795	.0796

THE LJUNG-BOX Q VALUE IS 46.78 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (21)
THE PIERCE QS VALUE IS 10.46 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (2)

BACKWARD RESIDUALS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	-.002	.002	.002	-.003	.004	.005	-.001	.000	-.003	-.001	.000	-.004
1973	-.007	-.001	-.004	-.002	-.005	.000	-.004	-.003	-.003	-.003	.002	.002
1974	-.002	.000	-.003	.000	.001	.005	.005	.004	-.002	-.002	.004	-.003
1975	.003	-.002	.004	.002	-.002	.003	-.003	-.001	.004	-.003	.002	-.001
1976	.002	.002	.006	-.001	.000	.004	-.001	.004	-.002	-.005	.003	-.005
1977	.000	.003	.002	.001	-.002	.004	-.004	.000	.004	.000	.001	.001
1978	.002	.002	.001	-.006	-.003	-.004	.001	-.003	.002	-.001	.002	-.006
1979	-.003	-.001	-.001	.003	-.002	-.005	.008	.005	.000	-.001	-.008	-.002
1980	.004	.001	-.003	.007	-.002	.000	-.003	.002	.001	-.002	.002	.003
1981	.003	.002	.001	.000	.004	-.001	-.008	.005	.000	-.002	-.003	-.002
1982	.002	-.001	-.004	-.005	.000	-.001	-.006	.001	-.003	-.004	.011	-.009
1983	.002	.001	-.003	-.004	.008	.000	-.004	.001	.001	-.001	.000	-.007
1984	.007	.004	.002	.003	.003	-.014	-.002	.002	-.002	.006	-.003	.002
1985	-.003	.002	.000	-.002	.000	.000	.003	-.001	.002	.001	.000	.002
1986	-.002	-.002	.004	.004	-.005	.001	.010	.000	.002	.002	-.004	.001
1987	.000	.000	.001	-.001	.002	.003	-.008	-.002	-.006	-.002	.000	-.002
1988	.003	-.001	.001	.001	.006	.004	-.003	.000	-.004	-.002	-.002	-.004
1989	-.002	.003	.000	-.003	-.001	-.004	.003	.000	.002	.003	.009	.008
1990	.005	.004	-.004	-.002	-.005	.001	.000	.000	.002	-.001	-.005	-.002
1991	-.003	-.004	.002	.002	.002	.000						

```
SECOND PART:
DERIVATION OF THE MODELS FOR THE COMPONENT

SERIES TITLE: alp234in

MODEL PARAMETERS
(0,1,1)(0,1,1)

PARAMETER VALUES PASSED FROM ARIMA ESTIMATION (TRUE SIGNS)

THETA PARAMETERS
 1.00    .26

BTHETA PARAMETERS
 1.00    .00    .00    .00    .00    .00    .00    .00    .00    .00    .00    .00   -.69

PHI PARAMETERS
 1.00

BPHI PARAMETERS
 1.00

NUMERATOR OF THE MODEL
 1.0000   .2602   .0000   .0000   .0000   .0000   .0000   .0000   .0000   .0000   .0000   .0000
 -.6855  -.1784

STATIONARY AUTOREGRESSIVE TREND
 1.0000

NON-STATIONARY AUTOREGRESSIVE TREND
 1.0000 -2.0000  1.0000

AUTOREGRESSIVE TREND
 1.0000 -2.0000  1.0000

STATIONARY AUTOREGRESSIVE CYCLE
 1.0000

NON-STATIONARY AUTOREGRESSIVE CYCLE
 1.0000

AUTOREGRESSIVE CYCLE
 1.0000

STATIONARY AUTOREGRESSIVE SEASONAL COMPONENT
 1.0000

NON-STATIONARY AUTOREGRESSIVE SEASONAL COMPONENT
 1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000
```

```
AUTOREGRESSIVE SEASONAL COMPONENT
 1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000  1.0000

STATIONARY AUTOREGRESSIVE SEASONALLY ADJUSTED COMPONENT
 1.0000

NON-STATIONARY AUTOREGRESSIVE SEASONALLY ADJUSTED COMPONENT
 1.0000 -2.0000  1.0000

AUTOREGRESSIVE SEASONALLY ADJUSTED COMPONENT
 1.0000 -2.0000  1.0000

TOTAL DENOMINATOR
 1.0000 -1.0000   .0000   .0000   .0000   .0000   .0000   .0000   .0000   .0000   .0000   .0000
-1.0000  1.0000
```

HARMONIC FUNCTIONS

$$\frac{F(X)}{H(X)} = \frac{F(X)}{T(X)\ C(X)\ S(X)}$$

F(X)

```
  1.5694     .7650     .0000     .0000     .0000     .0000     .0000     .0000     .0000     .0000     .0000    -.3568
 -1.4638    -.3568
```

T(X)

```
  6.0000   -8.0000    2.0000
```

C(X)

```
  1.0000
```

S(X)

```
 12.0000   22.0000   20.0000   18.0000   16.0000   14.0000   12.0000   10.0000    8.0000    6.0000    4.0000    2.0000
```

N(X), FORMED FROM THE PRODUCT T(S)C(X)

```
  6.0000   -8.0000    2.0000
```

H(X), FORMED FROM THE PRODUCT T(X)C(X)S(X) = N(X)S(X)

```
  4.0000   -4.0000     .0000     .0000     .0000     .0000     .0000     .0000     .0000     .0000     .0000    2.0000
 -4.0000    2.0000
```

```
F(X) = QT(X) + RT(X)
____           ____
H(X)           H(X)

QT(X) QUOTIENT
     -.1784

RT(X)  REMAINDER
     2.2829       .0515       .0000       .0000       .0000       .0000       .0000       .0000
     .0000       .0000       .0000       .0000      -2.1773


  RT(X)    = UT(X) +  V(X)
+________    _____    ____
T(X)S(X)     T(X)     S(X)

UT(X)
    2.2041    -2.2030

 V(X)
    1.4829     2.8333     2.5030     2.0691     1.6041     1.1611     .7753     .4670     .2433     .1004     .0256


DUM(X) =  U(X) - UT(X)C(X) - UC(X)T(X). THIS SHOULD BE ZERO

     .0000     .0000     .0000     .0000     .0000     .0000     .0000
     .0000     .0000     .0000     .0000     .0000     .0000
```

LOCAL MINIMA	FREQUENCY (RADIANS)	CONVERGENCE TEST
SEASONAL SPECTRUM.LOCAL MINIMA		
.092118	.0000	0
.034563	.8066	0
.010748	1.3241	0
.003854	1.8444	0
.001206	2.3647	0
.000221	2.8831	0
MINIMUM MINIMORUM		
.000221		
TREND SPECTRUM. SIMPLE MINIMUM		
.275441	3.1416	0

```
MODELS FOR THE COMPONENTS
-------------------------

TREND NUMERATOR
    1.0000      .0310     -.9690
TREND DENOMINATOR
    1.0000    -2.0000     1.0000
INNOV. VAR.      .28425


SEAS. NUMERATOR
    1.0000     2.0810     2.5901     2.7390     2.6056     2.2975     1.9024     1.4481     1.0370      .6144      .3373     -.0058
SEAS. DENOMINATOR
    1.0000     1.0000     1.0000     1.0000     1.0000     1.0000     1.0000     1.0000     1.0000     1.0000     1.0000     1.0000
INNOV. VAR.      .03806


IRREGULAR
VAR.       .09728


SEASONALLY ADJUSTED NUMERATOR
    1.0000     -.7145     -.2466

SEASONALLY ADJUSTED DENOMINATOR
    1.0000    -2.0000     1.0000
INNOV. VAR.      .72239
```

MA ROOTS OF TREND

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
.9690	.0000	.9690	.0000	-
-1.0000	.0000	1.0000	-180.0000	-2.0000

TOTAL SQUARED ERROR= .1350283D-30

MA ROOTS OF SEAS.

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
.5969	-.5968	.8441	-44.9963	-8.0006
.5969	.5968	.8441	44.9963	8.0006
-.6587	-.6482	.9241	-135.4593	-2.6576
-.6587	.6482	.9241	135.4593	2.6576
.2160	-.8285	.8562	-75.3904	-4.7751
.2160	.8285	.8562	75.3904	4.7751
-.2362	-.8509	.8831	-105.5145	-3.4119
-.2362	.8509	.8831	105.5145	3.4119
.0167	.0000	.0167	.0000	-
-.9668	-.2556	1.0000	-165.1919	-2.1793
-.9668	.2556	1.0000	165.1917	2.1793

TOTAL SQUARED ERROR= .1807077D-24

MA ROOTS OF SEASONALLY ADJUSTED SERIES

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
-.2545	.0000	.2545	180.0000	2.0
.9690	.0000	.9690	.0000	-

TOTAL SQUARED ERROR= .2445978D-31

AUTOCORRELATION FUNCTION OF COMPONENTS (STATIONARY TRANSFORMATION)

	TREND			ADJUSTED		
LAG	COMPONENT	ESTIMATOR	ESTIMATE	COMPONENT	ESTIMATOR	ESTIMATE
1	.000	.158	.154	-.343	-.346	-.365
2	-.500	-.539	-.443	-.157	-.153	-.131
3	.000	-.146	-.067	.000	.000	.112
4	.000	.038	-.088	.000	.000	-.129
5	.000	-.010	-.075	.000	.000	-.013
6	.000	.002	.006	.000	.000	.075
7	.000	.001	-.099	.000	.000	-.124
8	.000	-.006	.073	.000	.000	.039
9	.000	.023	.191	.000	.000	.139
10	.000	.085	.038	.000	.024	-.077
11	.000	-.025	-.012	.000	.054	.041
12	.000	-.157	-.111	.000	-.157	-.010
13	.000	-.025	-.162	.000	.054	-.149
14	.000	.085	-.014	.000	.024	.092
15	.000	.023	-.053	.000	.000	-.043
16	.000	-.006	-.045	.000	.000	-.077
17	.000	.002	.148	.000	.000	.133
18	.000	-.001	.059	.000	.000	.000
19	.000	.001	-.029	.000	.000	-.059
20	.000	-.004	.104	.000	.000	.079
21	.000	.016	.042	.000	.000	.045
22	.000	.058	-.029	.000	.016	-.121
23	.000	-.017	-.019	.000	.037	.154
24	.000	-.108	-.112	.000	-.108	-.186
VAR.(*)	.551	.314	.274	1.135	.947	.865

(*) IN UNITS OF VAR(A)

AUTOCORRELATION FUNCTION OF COMPONENTS (STATIONARY TRANSFORMATION)

	IRREGULAR			SEASONAL		
LAG	COMPONENT	ESTIMATOR	ESTIMATE	COMPONENT	ESTIMATOR	ESTIMATE
1	.000	-.630	-.630	.955	.845	.822
2	.000	.164	.131	.844	.482	.469
3	.000	-.043	.073	.698	.050	.093
4	.000	.011	-.107	.541	-.348	-.229
5	.000	-.003	.005	.391	-.636	-.422
6	.000	.001	.085	.261	-.769	-.502
7	.000	.000	-.110	.157	-.731	-.492
8	.000	-.002	.023	.082	-.534	-.394
9	.000	.007	.105	.033	-.217	-.223
10	.000	-.026	-.109	.008	.156	-.032
11	.000	.099	.056	.000	.493	.163
12	.000	-.157	.028	.000	.679	.269
13	.000	.099	-.135	.000	.621	.166
14	.000	-.026	.121	.000	.396	.000
15	.000	.007	-.039	.000	.105	-.131
16	.000	-.002	-.070	.000	-.174	-.185
17	.000	.000	.110	.000	-.384	-.143
18	.000	.000	-.027	.000	-.490	-.066
19	.000	.000	-.042	.000	-.478	.008
20	.000	-.001	.033	.000	-.354	.072
21	.000	.005	.074	.000	-.144	.129
22	.000	-.018	-.181	.000	.108	.169
23	.000	.068	.238	.000	.338	.215
24	.000	-.108	-.233	.000	.466	.219
VAR.(*)	.097	.030	.028	1.480	.071	.035

(*) IN UNITS OF VAR(A)

CROSSCORRELATION FUNCTION

	P(T)S(T-K)		S(T)U(T-K)		P(T)U(T-K)	
LAG	ESTIMATOR	ESTIMATE	ESTIMATOR	ESTIMATE	ESTIMATOR	ESTIMATE
-12	.148	.170	.042	.058	-.013	.111
-11	.094	.065	.034	.030	.102	.022
-10	-.035	-.126	.027	.028	-.013	-.029
-9	-.094	-.202	.019	.034	-.048	.050
-8	-.076	-.181	.011	.002	.012	-.112
-7	-.057	-.091	.003	-.016	-.003	-.039
-6	-.045	.006	-.006	-.018	.000	.102
-5	-.039	.020	-.015	-.056	.001	-.059
-4	-.039	.021	-.024	-.059	-.005	.009
-3	-.045	.002	-.033	-.028	.021	.124
-2	-.057	-.099	-.046	-.057	-.079	-.154
-1	-.076	-.185	-.044	-.042	.304	.265
0	-.094	-.208	.027	.067	.085	.116
1	-.035	-.133	.030	.050	-.652	-.667
2	.094	.060	.020	.060	.085	.114
3	.148	.161	.016	.032	.304	.267
4	.120	.155	.012	-.015	-.079	-.152
5	.085	.187	.010	-.003	.021	.119
6	.053	.180	.010	.000	-.005	.008
7	.021	.109	.012	-.015	.001	-.060
8	-.009	.054	.016	.028	.000	.101
9	-.039	-.013	.020	.062	-.003	-.033
10	-.067	-.086	.030	.048	.012	-.114
11	-.095	-.107	.027	.063	-.048	.045
12	-.122	-.131	-.044	-.038	-.013	-.026

THIRD PART:
ERROR ANALYSIS

ACF (LAG)	FINAL ESTIMATION ERROR		REVISION IN CONCURRENT ESTIMATORS	
	TREND	ADJUSTED	TREND	ADJUSTED
1	.664	.683	.607	.682
2	.214	.270	.334	.304
3	-.023	-.051	.040	-.002
4	-.221	-.282	-.169	-.235
5	-.338	-.425	-.318	-.392
6	-.387	-.483	-.398	-.471
7	-.366	-.458	-.409	-.469
8	-.280	-.352	-.347	-.383
9	-.130	-.170	-.211	-.212
10	.087	.088	-.004	.047
11	.341	.418	.302	.396
12	.480	.687	.537	.688
VAR.(*)	.212	.175	.213	.183

TOTAL ESTIMATION ERROR (CONCURRENT ESTIMATOR)

ACF (LAG)	TREND	ADJUSTED
1	.635	.683
2	.274	.288
3	.009	-.026
4	-.195	-.258
5	-.328	-.408
6	-.393	-.477
7	-.388	-.463
8	-.314	-.368
9	-.171	-.191
10	.041	.067
11	.322	.406
12	.508	.687
VAR.(*)	.426	.359

(*) IN UNITS OF VAR(A)

VARIANCE OF THE REVISION ERROR (*)

ADDITIONAL PERIODS	TREND	ADJUSTED
0	.2134	.1833
12	.0723	.0867
24	.0340	.0407
36	.0160	.0191
48	.0075	.0090
60	.0035	.0042

PERCENTAGE REDUCTION IN THE STANDARD ERROR OF THE REVISION AFTER ADDITIONAL YEARS
(COMPARISON WITH CONCURRENT ESTIMATORS)

AFTER 1 YEAR	41.8027	31.2325
AFTER 2 YEAR	60.1062	52.8604
AFTER 3 YEAR	72.6532	67.6863
AFTER 4 YEAR	81.2543	77.8495
AFTER 5 YEAR	87.1504	84.8166

VARIANCE OF THE REVISION ERROR FOR THE SEASONAL COMPONENT (ONE YEAR AHEAD ADJUSTMENT)

PERIODS AHEAD	VARIANCE (*)
0	.1833
1	.1946
2	.2187
3	.2337
4	.2417
5	.2449
6	.2454
7	.2455
8	.2474
9	.2531
10	.2649
11	.2850
12	.3156

AVERAGE PERCENTAGE REDUCTION IN RMSE FROM CONCURRENT ADJUSTMENT 12.2876

(*) IN UNITS OF VAR(A)

DECOMPOSITION OF THE SERIES: RECENT ESTIMATES

PERIOD	SERIES	TREND			ADJUSTED		
		ESTIMATE	STANDARD ERROR		ESTIMATE	STANDARD ERROR	
			TOTAL	DUE TO REVISION		TOTAL	DUE TO REVISION
-24	1.3627	1.3636	.0018	.0007	1.3634	.0017	.0007
-23	1.3865	1.3745	.0018	.0007	1.3755	.0017	.0008
-22	1.3844	1.3820	.0018	.0007	1.3812	.0017	.0008
-21	1.3897	1.3890	.0018	.0008	1.3896	.0017	.0008
-20	1.3934	1.3953	.0018	.0008	1.3951	.0017	.0009
-19	1.3891	1.3990	.0018	.0008	1.3997	.0017	.0009
-18	1.4030	1.4000	.0019	.0008	1.3998	.0017	.0009
-17	1.4055	1.4014	.0019	.0008	1.4010	.0017	.0009
-16	1.4005	1.4060	.0019	.0008	1.4059	.0018	.0009
-15	1.4107	1.4141	.0019	.0008	1.4134	.0018	.0009
-14	1.4267	1.4252	.0019	.0009	1.4257	.0018	.0009
-13	1.4348	1.4375	.0019	.0009	1.4370	.0018	.0010
-12	1.4495	1.4491	.0019	.0010	1.4500	.0019	.0011
-11	1.4688	1.4582	.0019	.0010	1.4577	.0019	.0011
-10	1.4697	1.4664	.0020	.0011	1.4666	.0019	.0012
-9	1.4744	1.4743	.0020	.0011	1.4745	.0020	.0012
-8	1.4791	1.4820	.0020	.0012	1.4815	.0020	.0013
-7	1.4818	1.4932	.0020	.0012	1.4929	.0020	.0013
-6	1.5100	1.5064	.0020	.0012	1.5069	.0020	.0013
-5	1.5231	1.5188	.0020	.0012	1.5185	.0020	.0013
-4	1.5268	1.5317	.0020	.0012	1.5317	.0020	.0013
-3	1.5422	1.5431	.0021	.0012	1.5439	.0020	.0013
-2	1.5519	1.5504	.0021	.0013	1.5503	.0020	.0014
-1	1.5540	1.5560	.0021	.0013	1.5560	.0021	.0014
0	1.5619	1.5626	.0024	.0017	1.5624	.0022	.0016
STANDARD ERROR OF FINAL ESTIMATOR			.0017			.0015	

PERIOD	SEASONAL		
	ESTIMATE	STANDARD ERROR	
		TOTAL	DUE TO REVISION
-24	-.0007	.0017	.0007
-23	.0110	.0017	.0008
-22	.0032	.0017	.0008
-21	.0002	.0017	.0008
-20	-.0018	.0017	.0009
-19	-.0106	.0017	.0009
-18	.0032	.0017	.0009
-17	.0044	.0017	.0009
-16	-.0054	.0018	.0009
-15	-.0027	.0018	.0009
-14	.0010	.0018	.0009
-13	-.0021	.0018	.0010
-12	-.0005	.0019	.0011
-11	.0111	.0019	.0011
-10	.0031	.0019	.0012
-9	-.0001	.0020	.0012
-8	-.0024	.0020	.0013
-7	-.0110	.0020	.0013
-6	.0032	.0020	.0013
-5	.0046	.0020	.0013
-4	-.0048	.0020	.0013
-3	-.0018	.0020	.0013
-2	.0016	.0020	.0014
-1	-.0019	.0021	.0014
0	-.0005	.0022	.0016
STANDARD ERROR OF FINAL ESTIMATOR		.0015	

DECOMPOSITION OF THE SERIES: FORECAST

PERIOD	SERIES		TREND			ADJUSTED		
	FORECAST	S.E.	FORECAST	STANDARD ERROR TOTAL	STANDARD ERROR DUE TO REVISION	FORECAST	STANDARD ERROR TOTAL	STANDARD ERROR DUE TO REVISION
1	1.5814	.0036	1.5705	.0037	.0033	1.5705	.0039	.0036
2	1.5818	.0058	1.5789	.0055	.0052	1.5789	.0056	.0054
3	1.5869	.0074	1.5872	.0068	.0066	1.5872	.0069	.0068
4	1.5929	.0087	1.5955	.0081	.0079	1.5955	.0081	.0080
5	1.5926	.0098	1.6038	.0092	.0090	1.6038	.0092	.0091
6	1.6151	.0108	1.6121	.0102	.0101	1.6121	.0103	.0101
7	1.6248	.0118	1.6203	.0112	.0111	1.6203	.0112	.0111
8	1.6236	.0126	1.6285	.0121	.0120	1.6285	.0122	.0121
9	1.6349	.0134	1.6366	.0131	.0130	1.6366	.0131	.0130
10	1.6464	.0142	1.6447	.0140	.0139	1.6447	.0140	.0139
11	1.6508	.0149	1.6528	.0149	.0148	1.6528	.0149	.0148
12	1.6603	.0156	1.6608	.0158	.0157	1.6608	.0158	.0157
13	1.6797	.0166	1.6688	.0166	.0165	1.6688	.0167	.0166
14	1.6797	.0176	1.6767	.0175	.0174	1.6767	.0175	.0175
15	1.6844	.0186	1.6847	.0184	.0183	1.6847	.0184	.0183
16	1.6899	.0196	1.6926	.0192	.0191	1.6926	.0192	.0192
17	1.6892	.0205	1.7004	.0201	.0200	1.7004	.0201	.0200
18	1.7113	.0213	1.7082	.0209	.0209	1.7082	.0210	.0209
19	1.7205	.0222	1.7160	.0218	.0217	1.7160	.0218	.0218
20	1.7189	.0230	1.7237	.0226	.0226	1.7237	.0227	.0226
21	1.7298	.0237	1.7314	.0235	.0234	1.7314	.0235	.0235
22	1.7408	.0245	1.7391	.0243	.0243	1.7391	.0244	.0243
23	1.7448	.0252	1.7467	.0252	.0251	1.7467	.0252	.0252
24	1.7538	.0259	1.7543	.0261	.0260	1.7543	.0261	.0260

PERIOD	SEASONAL		
	FORECAST	STANDARD ERROR	
		TOTAL	DUE TO REVISION
1	.0109	.0022	.0016
2	.0029	.0023	.0017
3	-.0003	.0023	.0018
4	-.0026	.0023	.0018
5	-.0112	.0023	.0018
6	.0031	.0023	.0018
7	.0045	.0023	.0018
8	-.0048	.0024	.0018
9	-.0017	.0024	.0018
10	.0017	.0024	.0019
11	-.0019	.0025	.0019
12	-.0005	.0025	.0020
13	.0109	.0026	.0021
14	.0029	.0026	.0021
15	-.0003	.0027	.0022
16	-.0026	.0027	.0022
17	-.0112	.0027	.0022
18	.0031	.0027	.0022
19	.0045	.0027	.0022
20	-.0048	.0027	.0022
21	-.0017	.0027	.0022
22	.0017	.0027	.0023
23	-.0019	.0028	.0023
24	-.0005	.0029	.0024

CONFIDENCE INTERVAL AROUND A SEASONAL FACTOR OF 100

	FINAL ESTIMATOR		CONCURRENT ESTIMATOR	
95% CONFIDENCE INTERVAL	99.7031	100.2977	99.5758	100.4261
70% CONFIDENCE INTERVAL	99.8428	100.1574	99.7753	100.2252

SAMPLE MEANS

	COMPLETE PERIOD	LAST THREE YEARS
SERIES	.2462	1.4066
TREND	.2462	1.4066
ADJUSTED	.2462	1.4066
SEASONAL	.0000	.0000

STANDARD ERROR OF THE RATES OF GROWTH ESTIMATES
(IN POINTS OF ANNUALIZED PERCENT GROWTH)
(LINEAR APPROXIMATION)

1. MONTHLY RATE OF THE MONTHLY SERIES (T11)

	TREND	SEASONALLY ADJ. SERIES
CONCURRENT ESTIMATOR	2.084	1.951
1 - PERIOD REVISION	1.926	1.947
2 - PERIOD REVISION	1.926	1.942
3 - PERIOD REVISION	1.918	1.938
4 - PERIOD REVISION	1.915	1.933
5 - PERIOD REVISION	1.910	1.930
6 - PERIOD REVISION	1.907	1.926
7 - PERIOD REVISION	1.903	1.922
8 - PERIOD REVISION	1.900	1.919
9 - PERIOD REVISION	1.897	1.916
10 - PERIOD REVISION	1.894	1.913
11 - PERIOD REVISION	1.874	1.908
12 - PERIOD REVISION	1.789	1.704
FINAL ESTIMATOR	1.642	1.449

2. MONTHLY RATE OF A 3 - MONTH MOVING AVERAGE (CENTERED) (T31)

	TREND	SEASONALLY ADJ. SERIES
CONCURRENT ESTIMATOR	1.595	1.658
1 - PERIOD REVISION	1.247	1.196
2 - PERIOD REVISION	1.208	1.189
3 - PERIOD REVISION	1.205	1.181
4 - PERIOD REVISION	1.196	1.175
5 - PERIOD REVISION	1.190	1.168
6 - PERIOD REVISION	1.184	1.162
7 - PERIOD REVISION	1.178	1.156
8 - PERIOD REVISION	1.173	1.150
9 - PERIOD REVISION	1.168	1.145
10 - PERIOD REVISION	1.168	1.145
11 - PERIOD REVISION	1.142	1.106
12 - PERIOD REVISION	1.098	1.064
FINAL ESTIMATOR	.955	.880

3. ACCUMULATED RATE OVER THE LAST DECEMBER

	CONCURRENT ESTIMATOR		FINAL ESTIMATOR	
	TREND	SEONALLY ADJ. SERIES	TREND	SEASONALLY ADJ. SERIES
JANUARY	2.084	1.951	1.642	1.449
FEBRUARY	1.577	1.485	1.256	1.100
MARCH	1.247	1.196	.955	.880
APRIL	1.036	.998	.783	.729
MAY	.878	.847	.655	.615
JUNE	.751	.724	.556	.522
JULY	.643	.617	.473	.444
AUGUST	.545	.521	.401	.374
SEPTEMBER	.453	.428	.335	.309
OCTOBER	.361	.335	.271	.246
NOVEMBER	.262	.229	.209	.179
DECEMBER	.191	.129	.170	.120

(CENTERED) ESTIMATOR OF THE PRESENT
RATE OF ANNUAL GROWTH, T(1 MQ)
(LINEAR APPROXIMATION)

STANDARD ERROR	TREND	SEAS. ADJ. SERIES	ORIGINAL SERIES
CONCURRENT ESTIMATOR	1.065	1.070	1.083
FINAL ESTI-MATOR	.170	.120	.000

FOURTH PART:
ESTIMATES OF THE COMPONENTS

ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	.25	.25	.25	.25	.26	.26	.27	.27	.28	.28	.28	.29
1973	.30	.30	.31	.32	.32	.33	.34	.34	.35	.35	.36	.37
1974	.38	.38	.38	.39	.40	.40	.41	.41	.42	.42	.43	.44
1975	.45	.45	.46	.46	.47	.47	.49	.50	.50	.50	.51	.53
1976	.54	.54	.54	.55	.56	.56	.58	.59	.59	.60	.61	.62
1977	.64	.64	.65	.66	.66	.67	.70	.70	.71	.72	.72	.74
1978	.76	.75	.76	.78	.79	.81	.83	.84	.85	.86	.86	.89
1979	.91	.91	.93	.94	.95	.97	1.00	1.00	1.00	1.02	1.02	1.06
1980	1.08	1.07	1.09	1.11	1.11	1.13	1.16	1.18	1.18	1.20	1.20	1.24
1981	1.25	1.25	1.27	1.29	1.30	1.31	1.35	1.37	1.37	1.39	1.40	1.44
1982	1.47	1.47	1.49	1.52	1.54	1.56	1.60	1.62	1.63	1.66	1.66	1.70
1983	1.74	1.73	1.76	1.79	1.80	1.81	1.86	1.87	1.89	1.91	1.91	1.97
1984	2.00	1.99	2.01	2.03	2.04	2.07	2.14	2.15	2.17	2.19	2.18	2.25
1985	2.27	2.28	2.31	2.34	2.36	2.39	2.44	2.44	2.46	2.48	2.49	2.55
1986	2.57	2.58	2.62	2.64	2.66	2.70	2.74	2.73	2.75	2.77	2.78	2.85
1987	2.89	2.89	2.93	2.97	2.99	3.01	3.08	3.11	3.14	3.18	3.19	3.27
1988	3.31	3.31	3.35	3.39	3.41	3.43	3.49	3.51	3.53	3.57	3.58	3.67
1989	3.72	3.73	3.76	3.82	3.85	3.91	4.00	3.99	4.01	4.03	4.01	4.07
1990	4.08	4.06	4.10	4.16	4.20	4.26	4.34	4.35	4.37	4.39	4.40	4.53
1991	4.59	4.60	4.67	4.72	4.73	4.77						

TRANSFORMED SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	-1.41	-1.40	-1.39	-1.37	-1.35	-1.34	-1.31	-1.30	-1.29	-1.27	-1.26	-1.23
1973	-1.20	-1.19	-1.17	-1.15	-1.13	-1.11	-1.08	-1.07	-1.06	-1.04	-1.02	-1.00
1974	-.97	-.97	-.96	-.94	-.93	-.91	-.89	-.88	-.88	-.86	-.85	-.82
1975	-.80	-.80	-.78	-.77	-.76	-.74	-.72	-.70	-.69	-.68	-.67	-.64
1976	-.62	-.62	-.61	-.60	-.59	-.57	-.54	-.53	-.53	-.51	-.50	-.47
1977	-.45	-.45	-.44	-.42	-.41	-.39	-.36	-.35	-.34	-.33	-.33	-.30
1978	-.28	-.28	-.27	-.25	-.23	-.22	-.18	-.17	-.16	-.15	-.15	-.12
1979	-.09	-.09	-.08	-.06	-.05	-.03	.00	.00	.00	.02	.02	.06
1980	.08	.07	.09	.10	.11	.12	.15	.16	.17	.18	.19	.21
1981	.23	.22	.24	.25	.26	.27	.30	.31	.32	.33	.34	.37
1982	.38	.38	.40	.42	.43	.44	.47	.48	.49	.51	.51	.53
1983	.55	.55	.56	.58	.59	.59	.62	.63	.64	.64	.65	.68
1984	.69	.69	.70	.71	.71	.73	.76	.77	.77	.78	.78	.81
1985	.82	.83	.84	.85	.86	.87	.89	.89	.90	.91	.91	.94
1986	.95	.95	.96	.97	.98	.99	1.01	1.00	1.01	1.02	1.02	1.05
1987	1.06	1.06	1.07	1.09	1.09	1.10	1.12	1.13	1.14	1.16	1.16	1.19
1988	1.20	1.20	1.21	1.22	1.23	1.23	1.25	1.25	1.26	1.27	1.28	1.30
1989	1.31	1.32	1.33	1.34	1.35	1.36	1.39	1.38	1.39	1.39	1.39	1.40
1990	1.41	1.40	1.41	1.43	1.43	1.45	1.47	1.47	1.47	1.48	1.48	1.51
1991	1.52	1.53	1.54	1.55	1.55	1.56						

SEASONAL FACTORS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	101.32	99.93	99.74	99.89	99.62	99.64	100.86	100.38	99.56	99.55	99.13	100.38
1973	101.30	99.89	99.72	99.87	99.58	99.64	100.90	100.42	99.61	99.57	99.16	100.43
1974	101.32	99.85	99.70	99.84	99.54	99.62	100.93	100.44	99.62	99.56	99.15	100.46
1975	101.34	99.83	99.67	99.79	99.48	99.57	100.96	100.50	99.69	99.58	99.14	100.49
1976	101.34	99.81	99.64	99.78	99.45	99.55	101.01	100.53	99.73	99.60	99.10	100.49
1977	101.30	99.76	99.61	99.79	99.46	99.54	101.08	100.58	99.80	99.64	99.06	100.48
1978	101.22	99.68	99.57	99.82	99.46	99.53	101.13	100.59	99.83	99.67	99.03	100.49
1979	101.17	99.66	99.60	99.88	99.47	99.50	101.13	100.57	99.84	99.70	99.02	100.51
1980	101.11	99.62	99.60	99.92	99.49	99.46	101.09	100.57	99.90	99.78	99.05	100.50
1981	101.04	99.59	99.60	99.93	99.53	99.44	101.05	100.55	99.93	99.82	99.04	100.46
1982	100.97	99.58	99.62	99.95	99.58	99.48	101.04	100.50	99.96	99.86	99.02	100.43
1983	100.92	99.60	99.66	99.97	99.59	99.51	101.04	100.43	99.98	99.87	98.98	100.44
1984	100.86	99.62	99.70	99.97	99.58	99.57	101.06	100.37	99.98	99.86	98.95	100.43
1985	100.76	99.63	99.76	100.02	99.65	99.70	101.04	100.28	99.95	99.83	98.93	100.42
1986	100.68	99.62	99.81	100.07	99.70	99.77	101.01	100.23	99.93	99.82	98.94	100.42
1987	100.64	99.60	99.80	100.09	99.73	99.78	100.96	100.22	99.95	99.84	98.98	100.42
1988	100.61	99.57	99.79	100.11	99.77	99.83	100.99	100.24	99.95	99.82	98.95	100.35
1989	100.52	99.51	99.75	100.10	99.79	99.93	101.10	100.32	100.01	99.82	98.95	100.32
1990	100.44	99.46	99.73	100.10	99.78	99.95	101.11	100.31	99.99	99.76	98.90	100.32
1991	100.46	99.51	99.82	100.16	99.80	99.94	101.09	100.29	99.97	99.73	98.88	100.31
1992	100.45	99.52	99.83	100.17	99.81	99.94	101.09	100.29	99.97	99.73	98.88	100.31
1993	100.45	99.52	99.83	100.17	99.81	99.94						

BIAS CORRECTION = .00 PERCENT

SEASONAL COMPONENT (LEVELS)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
1973	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
1974	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
1975	.01	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
1976	.01	.00	.00	.00	.00	.00	.01	.00	.00	.00	-.01	.00
1977	.01	.00	.00	.00	.00	.00	.01	.00	.00	.00	-.01	.00
1978	.01	.00	.00	.00	.00	.00	.01	.00	.00	.00	-.01	.00
1979	.01	.00	.00	.00	-.01	.00	.01	.01	.00	.00	-.01	.01
1980	.01	.00	.00	.00	-.01	-.01	.01	.01	.00	.00	-.01	.01
1981	.01	-.01	-.01	.00	-.01	-.01	.01	.01	.00	.00	-.01	.01
1982	.01	-.01	-.01	.00	-.01	-.01	.02	.01	.00	.00	-.02	.01
1983	.02	-.01	-.01	.00	-.01	-.01	.02	.01	.00	.00	-.02	.01
1984	.02	-.01	-.01	.00	-.01	-.01	.02	.01	.00	.00	-.02	.01
1985	.02	-.01	-.01	.00	-.01	-.01	.03	.01	.00	.00	-.03	.01
1986	.02	-.01	-.01	.00	-.01	-.01	.03	.01	.00	.00	-.03	.01
1987	.02	-.01	-.01	.00	-.01	-.01	.03	.01	.00	.00	-.03	.01
1988	.02	-.01	-.01	.00	-.01	-.01	.03	.01	.00	-.01	-.04	.01
1989	.02	-.02	-.01	.00	-.01	.00	.04	.01	.00	-.01	-.04	.01
1990	.02	-.02	-.01	.00	-.01	.00	.05	.01	.00	-.01	-.05	.01
1991	.02	-.02	-.01	.01	-.01	.00						

TREND

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	.24	.25	.25	.25	.26	.26	.27	.27	.28	.28	.29	.29
1973	.30	.30	.31	.32	.32	.33	.34	.34	.35	.36	.36	.37
1974	.37	.38	.39	.39	.40	.40	.41	.41	.42	.42	.43	.44
1975	.44	.45	.46	.46	.47	.48	.48	.49	.50	.51	.51	.52
1976	.53	.54	.54	.55	.56	.57	.58	.58	.59	.60	.61	.62
1977	.63	.64	.65	.66	.67	.68	.69	.70	.71	.72	.73	.74
1978	.75	.76	.77	.78	.79	.81	.82	.84	.85	.86	.87	.89
1979	.90	.92	.93	.94	.96	.97	.99	.99	1.00	1.02	1.03	1.05
1980	1.07	1.08	1.09	1.11	1.12	1.13	1.15	1.17	1.19	1.20	1.22	1.23
1981	1.24	1.25	1.27	1.29	1.30	1.32	1.34	1.36	1.38	1.39	1.41	1.43
1982	1.45	1.47	1.50	1.52	1.55	1.57	1.59	1.61	1.64	1.66	1.68	1.69
1983	1.72	1.74	1.76	1.79	1.81	1.82	1.84	1.87	1.89	1.91	1.93	1.96
1984	1.98	2.00	2.02	2.03	2.05	2.08	2.12	2.14	2.17	2.19	2.21	2.24
1985	2.26	2.29	2.32	2.34	2.37	2.40	2.42	2.44	2.46	2.49	2.51	2.54
1986	2.56	2.59	2.62	2.64	2.67	2.70	2.72	2.73	2.75	2.78	2.81	2.84
1987	2.87	2.90	2.93	2.96	2.99	3.02	3.05	3.10	3.14	3.19	3.23	3.26
1988	3.29	3.32	3.36	3.39	3.42	3.44	3.46	3.50	3.53	3.58	3.62	3.66
1989	3.70	3.74	3.78	3.81	3.86	3.91	3.95	3.98	4.01	4.04	4.05	4.06
1990	4.06	4.08	4.11	4.16	4.21	4.26	4.30	4.33	4.37	4.40	4.45	4.51
1991	4.57	4.63	4.68	4.71	4.74	4.77						

SEASONALLY ADJUSTED SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	.24	.25	.25	.25	.26	.26	.27	.27	.28	.28	.29	.29
1973	.30	.30	.31	.32	.32	.33	.34	.34	.35	.36	.36	.37
1974	.37	.38	.39	.39	.40	.40	.41	.41	.42	.42	.43	.44
1975	.45	.45	.46	.46	.47	.48	.48	.49	.50	.51	.52	.52
1976	.53	.54	.54	.55	.56	.57	.57	.58	.59	.60	.61	.62
1977	.63	.64	.65	.66	.67	.68	.69	.70	.71	.72	.73	.74
1978	.75	.76	.77	.78	.80	.81	.82	.84	.85	.86	.87	.89
1979	.90	.92	.93	.95	.96	.97	.99	.99	1.00	1.02	1.03	1.05
1980	1.07	1.08	1.09	1.11	1.12	1.14	1.15	1.17	1.19	1.20	1.22	1.23
1981	1.24	1.25	1.27	1.29	1.30	1.32	1.34	1.36	1.37	1.39	1.41	1.43
1982	1.45	1.47	1.50	1.52	1.55	1.57	1.59	1.61	1.63	1.66	1.68	1.69
1983	1.72	1.74	1.76	1.79	1.81	1.82	1.84	1.87	1.89	1.91	1.93	1.96
1984	1.98	2.00	2.02	2.03	2.05	2.08	2.12	2.14	2.17	2.19	2.21	2.24
1985	2.26	2.29	2.32	2.34	2.37	2.40	2.42	2.43	2.46	2.49	2.51	2.54
1986	2.56	2.59	2.62	2.64	2.66	2.70	2.72	2.72	2.75	2.78	2.81	2.84
1987	2.87	2.90	2.93	2.96	3.00	3.02	3.05	3.10	3.14	3.19	3.23	3.26
1988	3.29	3.32	3.36	3.39	3.42	3.43	3.46	3.50	3.53	3.58	3.62	3.66
1989	3.70	3.75	3.77	3.81	3.86	3.91	3.96	3.98	4.01	4.04	4.05	4.05
1990	4.06	4.08	4.11	4.16	4.21	4.26	4.30	4.33	4.37	4.40	4.45	4.51
1991	4.57	4.63	4.68	4.71	4.74	4.77						

IRREGULAR FACTORS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1972	99.98	100.05	99.99	99.96	100.07	99.99	99.95	100.02	99.97	100.03	100.00	99.96
1973	99.99	100.05	99.96	100.04	99.94	100.06	99.97	100.02	100.00	99.99	100.05	100.00
1974	99.97	100.00	99.98	100.04	99.99	100.03	100.02	99.98	99.96	100.01	100.03	99.93
1975	100.10	99.90	100.10	99.95	99.97	100.03	99.96	100.02	100.07	99.90	100.05	99.99
1976	100.03	99.99	100.04	99.92	100.04	100.00	99.97	100.05	99.93	100.01	100.04	99.94
1977	100.06	100.01	99.96	100.02	99.97	100.03	99.95	100.04	100.04	99.98	99.97	100.04
1978	99.98	99.99	99.98	99.96	100.05	99.93	100.12	99.90	100.08	99.98	100.00	99.95
1979	100.02	100.01	99.99	100.07	99.93	99.97	100.15	99.93	99.96	100.04	99.88	100.13
1980	100.00	99.96	99.97	100.12	99.90	100.04	99.93	100.07	99.98	99.99	100.03	100.02
1981	99.95	100.01	100.01	99.96	100.09	99.92	99.94	100.12	99.92	100.03	99.98	100.02
1982	100.00	99.98	99.98	99.99	100.07	99.98	99.97	100.03	99.97	100.01	100.13	99.78
1983	100.14	99.96	99.99	99.99	100.11	99.89	100.02	99.98	100.04	99.98	99.97	99.99
1984	100.09	99.96	100.02	100.01	99.95	99.88	100.17	99.95	99.99	100.11	99.84	100.15
1985	99.84	100.10	99.97	99.99	99.99	100.07	100.00	99.92	100.07	99.97	100.00	100.05
1986	99.90	100.03	100.07	99.99	99.89	100.14	100.02	99.88	100.06	99.98	99.94	100.08
1987	99.95	100.01	100.00	99.99	100.04	100.01	99.85	100.12	99.93	100.06	100.02	99.96
1988	100.06	99.94	100.03	100.00	100.05	99.97	99.95	100.03	99.97	100.01	100.02	99.97
1989	100.01	100.06	99.95	99.99	100.00	99.98	100.11	99.93	100.06	99.98	100.07	99.98
1990	99.96	99.99	99.93	100.04	99.94	100.09	99.96	100.02	100.02	99.95	99.97	100.05
1991	99.97	100.00	100.08	99.99	100.00	99.98						

BIAS CORRECTION = .00 PERCENT

* *

* PROCESSING COMPLETED* *

* *

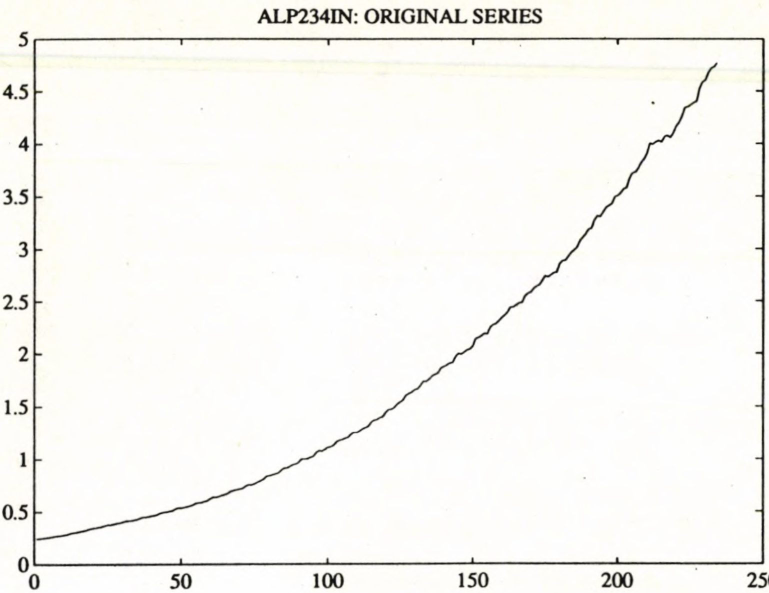

ALP234IN: ORIGINAL SERIES
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
50
100
150
200
250

Fig. 1

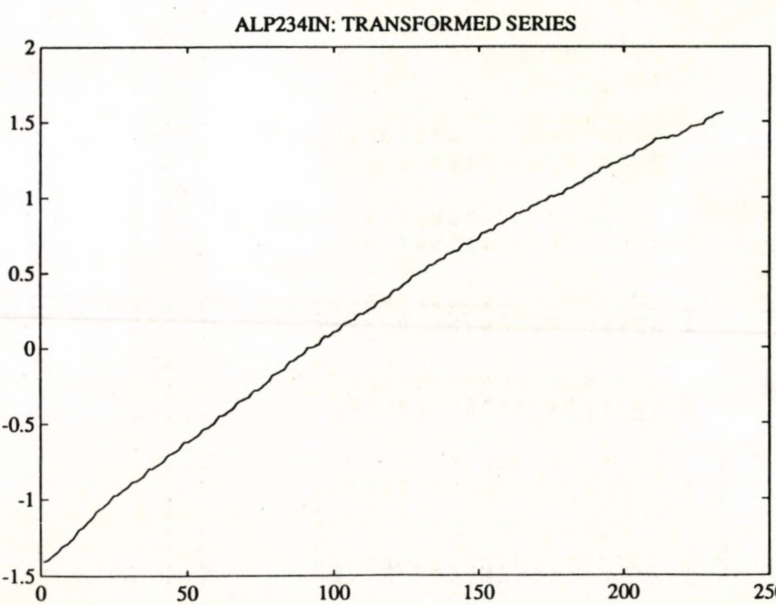

ALP234IN: TRANSFORMED SERIES
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
50
100
150
200
250

Fig. 2

ALP234IN: DIFFERENCED SERIES

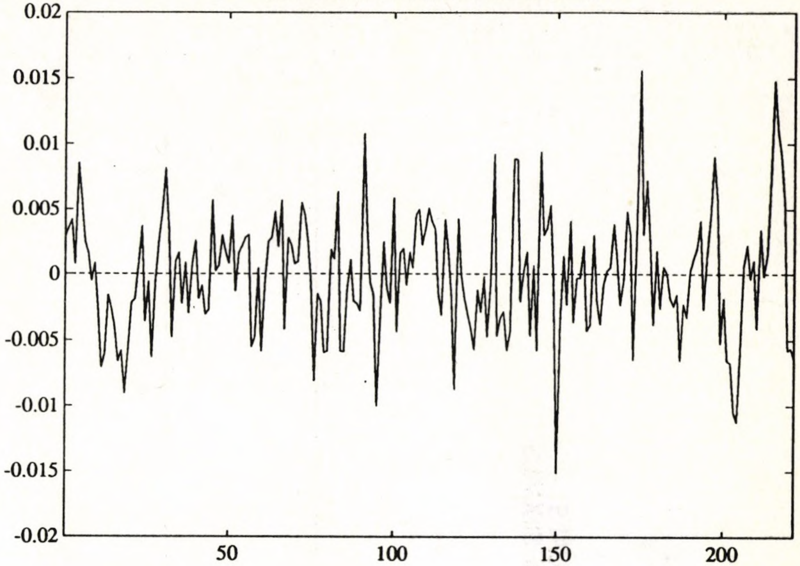


Fig. 3

ALP234IN: RESIDUALS

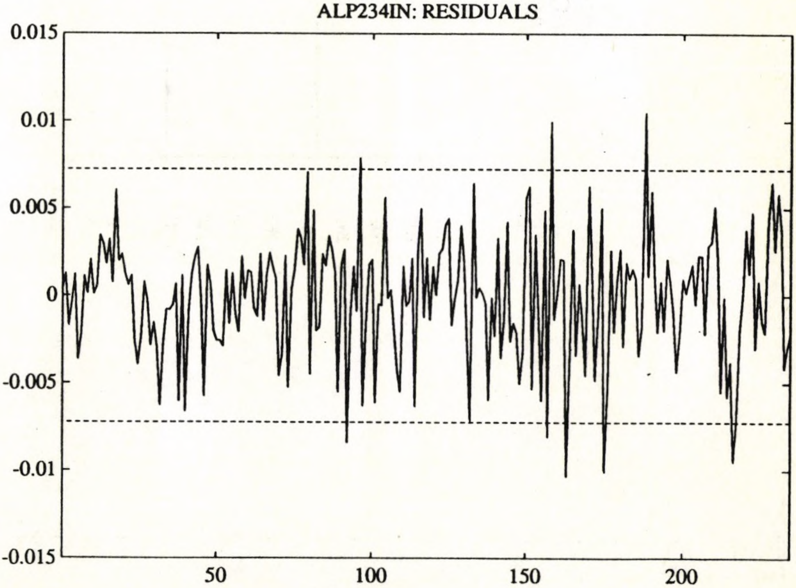
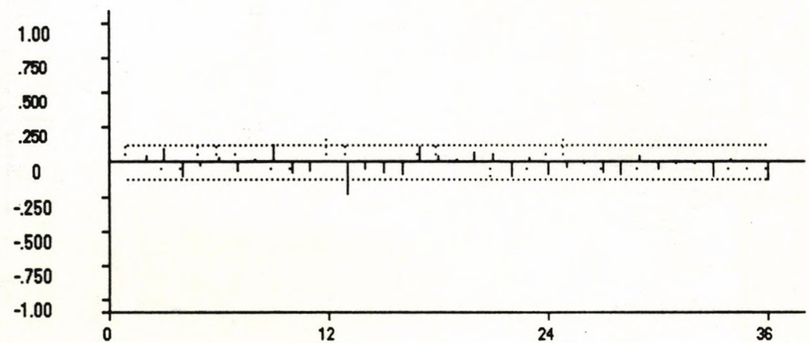


Fig. 4

ALP234IN:
ACF RESIDUALS
ACF SQD RESIDUALS



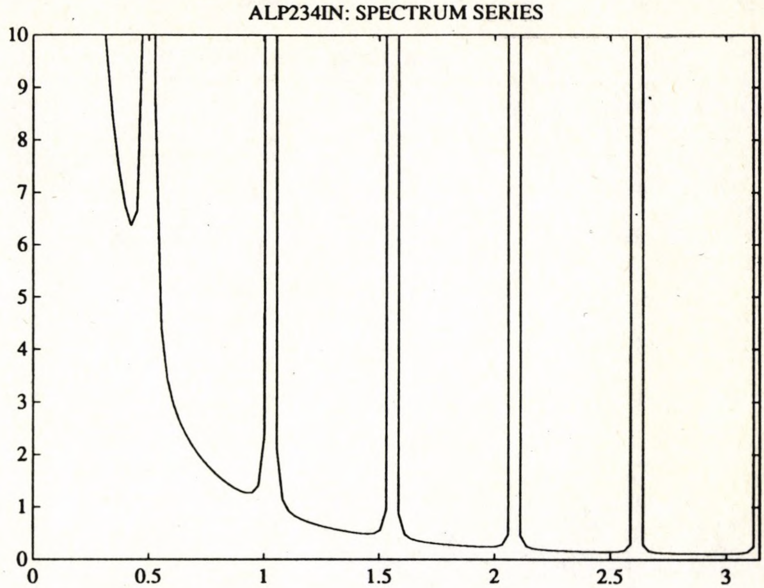


Fig. 5

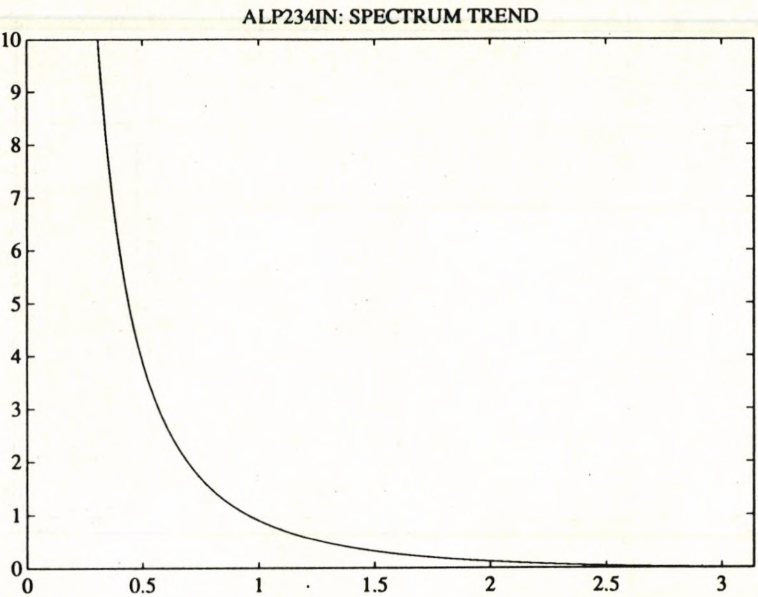

ALP234IN: SPECTRUM TREND
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3

Fig. 6

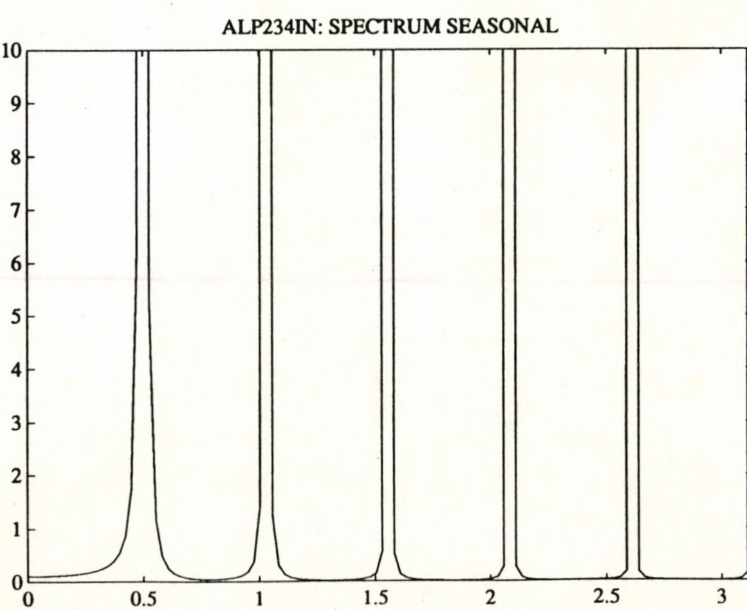
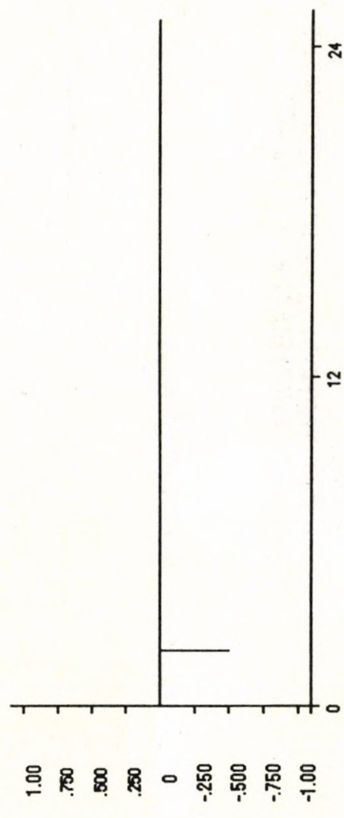
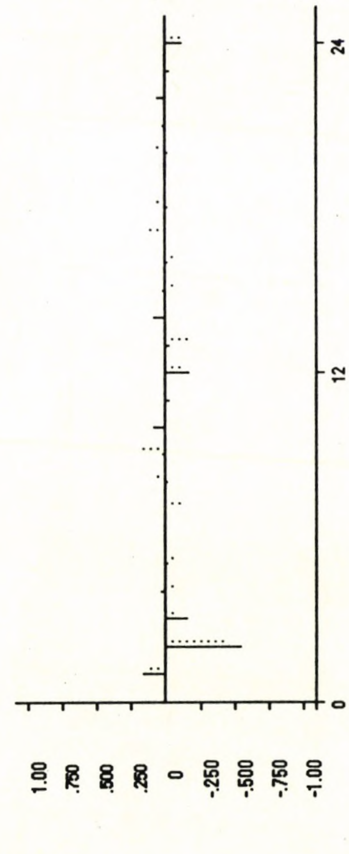

ALP234IN: SPECTRUM SEASONAL
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3

Fig. 7

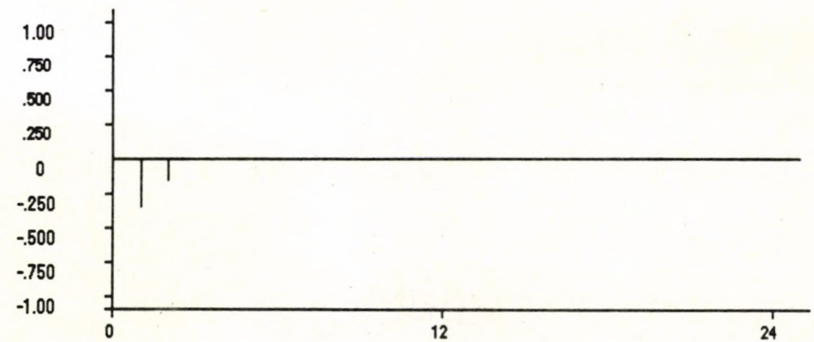
ALP234: ACF OF TREND TH. COMPONENT



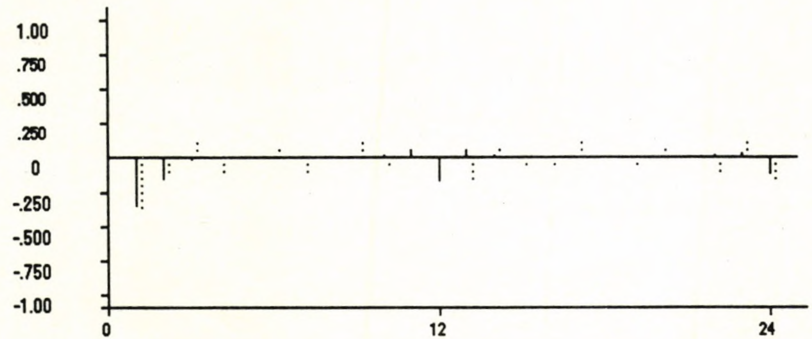
ALP234:
ACF OF TREND THEORETICAL ESTIMATOR
ACF OF TREND EMPIRICAL ESTIMATE



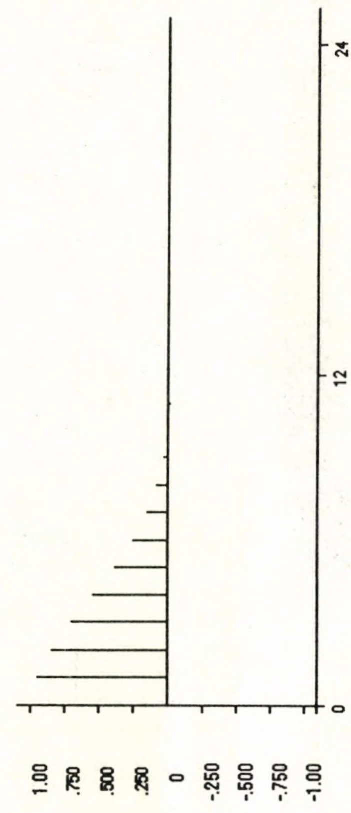
ALP234: ACF OF S. ADJ. TH. COMPONENT



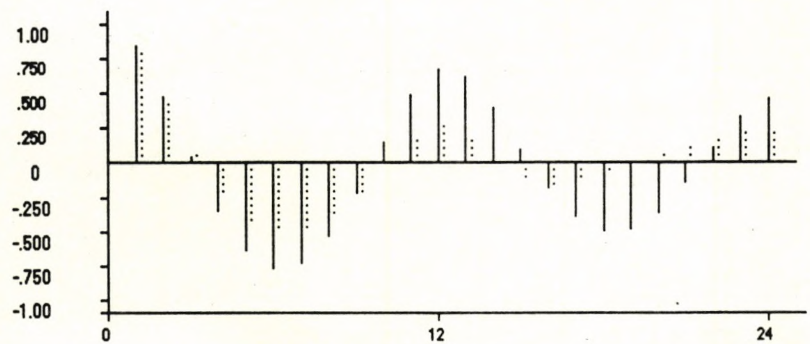
ALP234:
ACF OF S. ADJ. THEORETICAL ESTIMATOR
ACF OF S. ADJ. EMPIRICAL ESTIMATE



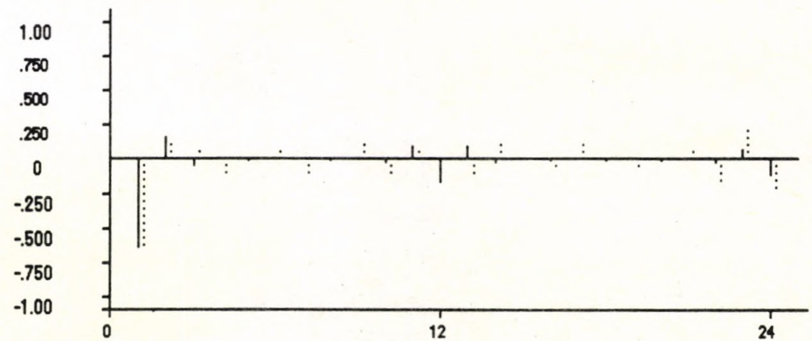
ALP234: ACF OF SEASONAL TH. COMPONENT

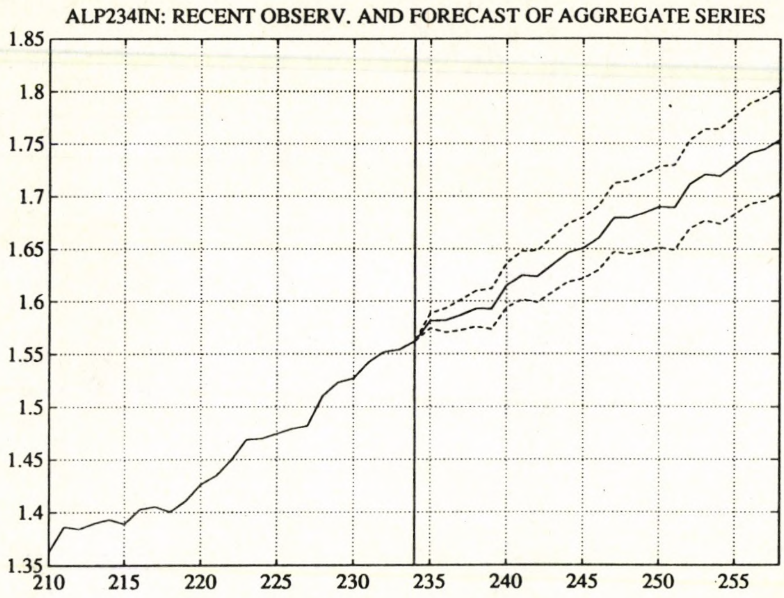


ALP234:
ACF OF SEASONAL THEORETICAL ESTIMATOR
ACF OF SEASONAL EMPIRICAL ESTIMATE

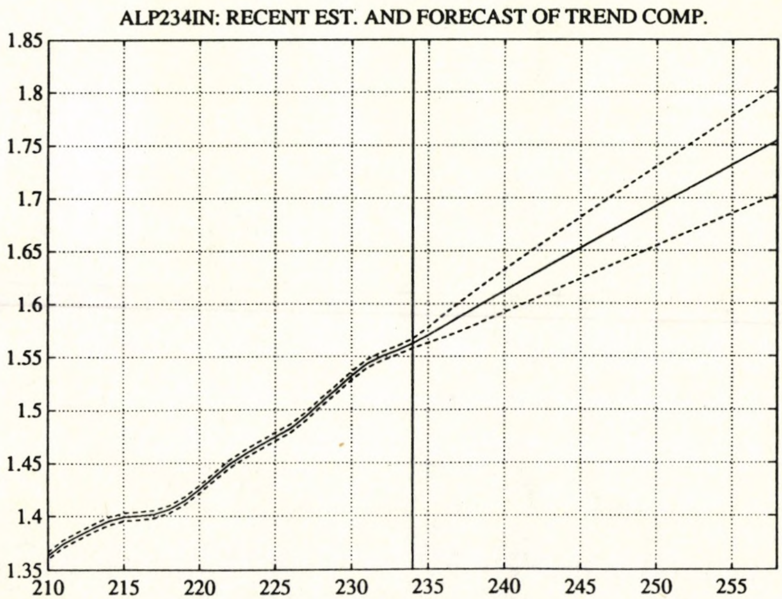


ALP234:
ACF OF IRR. THEORETICAL ESTIMATOR
ACF OF IRR. EMPIRICAL ESTIMATE



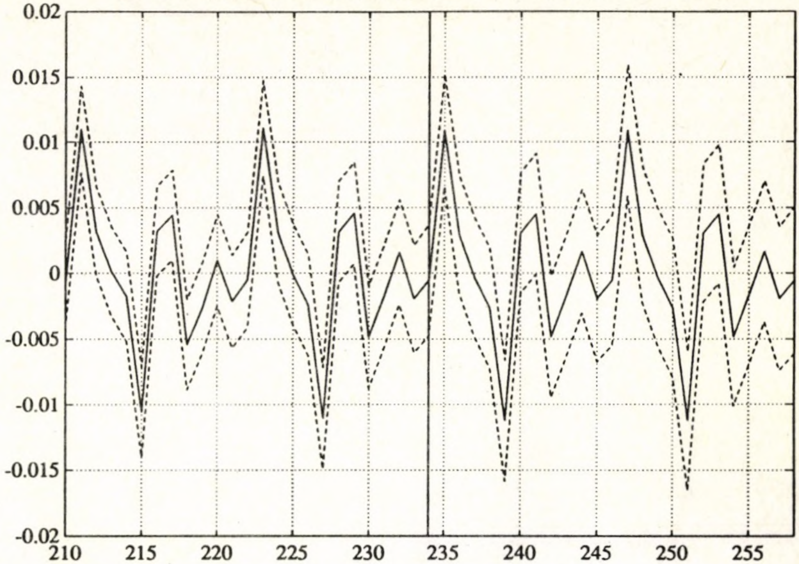


====: 95% C.I. Fig. 12



====: 95% C.I. Fig. 13

ALP234IN: RECENT EST. AND FORECAST OF SEASONAL COMP.



====: 95% C.I. Fig. 14

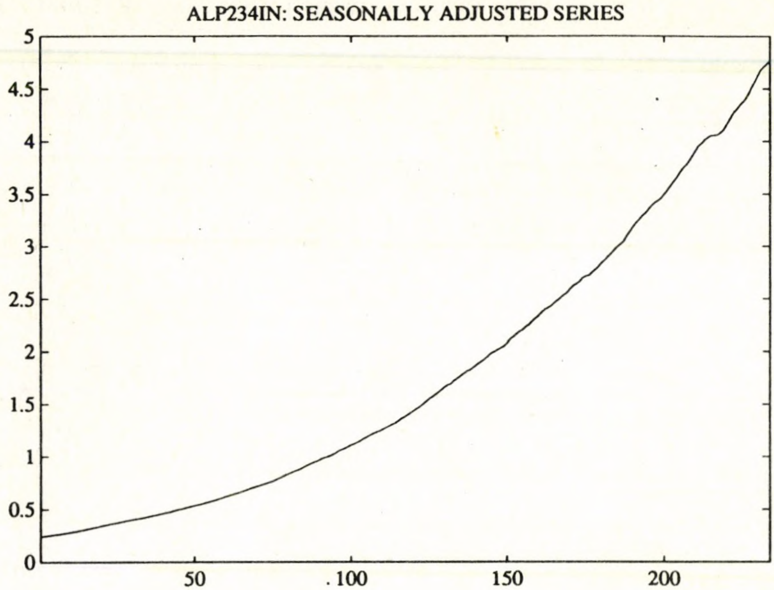

ALP234IN: SEASONALLY ADJUSTED SERIES
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
50
100
150
200

Fig. 8

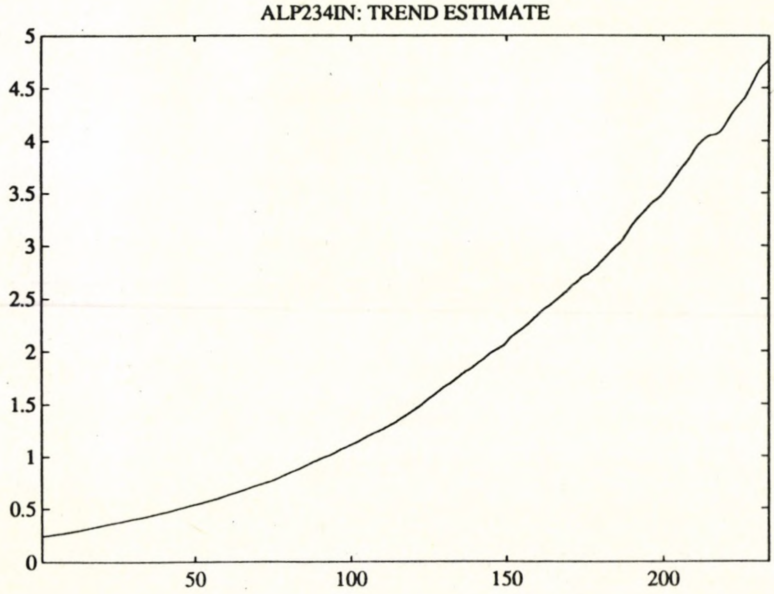

ALP234IN: TREND ESTIMATE
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
50
100
150
200

Fig. 9

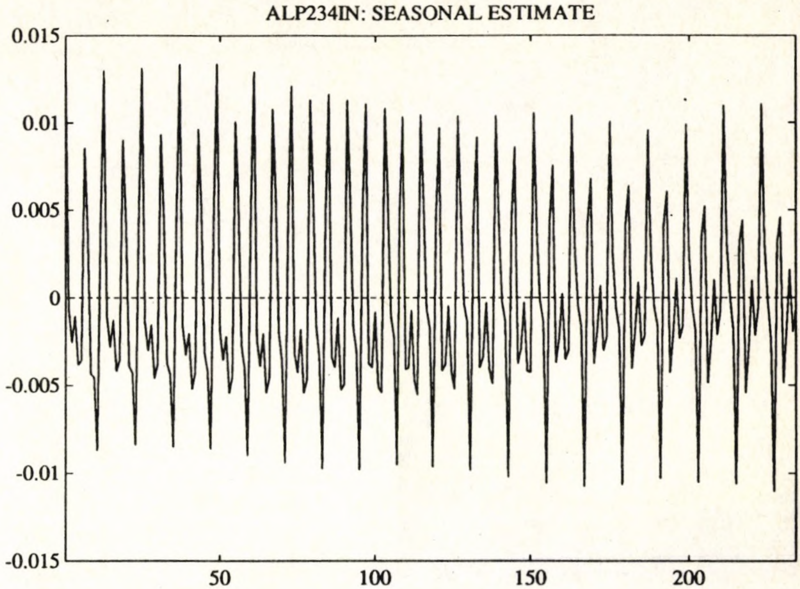

ALP234IN: SEASONAL ESTIMATE
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
50
100
150
200

Fig. 10

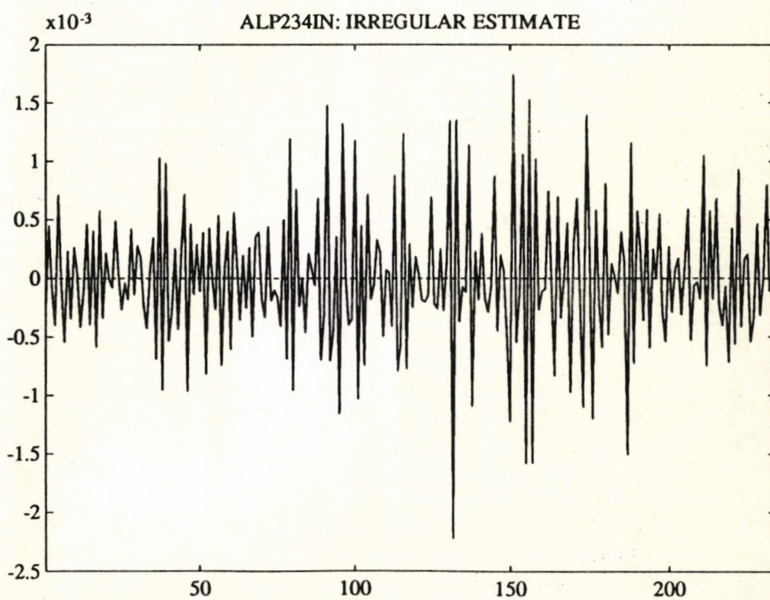

x10-3
ALP234IN: IRREGULAR ESTIMATE
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
50
100
150
200

Fig. 11

PROGRAM SEATS
(Signal Extraction in Arima Time Series)

6. Example 2: SIMULATED SERIES
(Nonstationary Trend, Seasonal and Cyclical Components)

This second printout presents the results for a simulated quarterly series with 80 observations (a fairly standard sample size). The model used to generate the series was the multiplicative ARIMA $(2, 1, 2) \times (0, 1, 0)_4$ in the levels:

$$\left(1 - \phi_1 B - \phi_1 B^2\right) \nabla \nabla_4 z_t = \left(1 - \theta_1 B - \theta_2 B^2\right) a_t ,$$

where the AR(2) polynomial is equal to

$$1 - 1.4B + B^2 ,$$

and the MA(2) polynomial is given by

$$1 - B + .24 B^2 .$$

The AR polynomial contains a complex root having period 7.9 quarters and modulus 1. Thus the AR(2) contains a pair of complex conjugate unit roots representing a nonstationary cycle of period close to 2 years.

Maximum likelihood estimation is seen to yield

$$\left(1 - 1.395 B + B^2\right) \nabla \nabla_4 z_t = \left(1 - 1.01 B + .27 B^2\right) a_t ,$$

and hence captures very well the unit root, i.e., the nonstatioary cycle. This cycle is then treated as a separate component, and the series is decomposed into trend, seasonal, irregular and cyclical components. The model for the cycle is derived in the manner explained in the paper "On the Dynamic Structure of a Seasonal Component."

In this example, the non-default parameter values are p=2, q=2, bq=0, mq=4, lam=1. Given the relatively large number of "regular" parameters and the relatively short length of the series, it is of interest to see how well the estimated model captures the theoretical one, and how close the decomposition and the estimates of the components are to their theoretical values.

SIGNAL EXTRACTION IN 'ARIMA' TIME SERIES

BY AGUSTIN MARAVALL & VICTOR GOMEZ, with the assistance of GABRIELE FIORENTINI and GIANLUCA CAPORELLO (VERSION 1991)

(Based on an original program developed by J.P. BURMAN at the Bank of England, version 1982)

FIRST PART:
ARIMA ESTIMATION

SERIES TITLE: SIMUL

METHOD - MAXIMUM LIKELIHOOD

NO OF OBSERVATIONS = 80

ORIGINAL SERIES

YEAR	1ST	2ND	3RD	4TH
1920	969.941	965.979	942.683	930.778
1921	943.355	954.446	953.613	956.102
1922	968.184	963.244	940.369	928.273
1923	941.261	952.060	952.432	955.133
1924	966.174	961.096	940.473	930.306
1925	946.122	957.329	954.891	955.537
1926	965.532	959.576	938.657	930.775
1927	946.097	957.892	955.427	956.267
1928	964.839	960.610	941.410	934.563
1929	948.617	959.596	956.813	957.952
1930	965.537	960.192	939.869	934.374
1931	948.750	960.103	955.732	958.097
1932	965.223	957.227	935.384	934.546
1933	952.424	963.357	956.548	958.783
1934	965.047	956.202	934.603	936.791
1935	956.879	968.868	961.440	964.226
1936	968.063	959.012	937.699	940.366
1937	961.709	973.938	965.999	965.382
1938	968.992	959.595	939.582	942.758
1939	964.026	975.768	966.675	965.206

TRANSFORMATION: Z -> Z

NONSEASONAL DIFFERENCING D= 1
SEASONAL DIFFERENCING BD= 1

DIFFERENCED SERIES

YEAR	1ST	2ND	3RD	4TH
1921		15.053	22.463	14.394
1922	-.495	-16.031	-22.042	-14.585
1923	.906	15.739	23.247	14.797
1924	-1.947	-15.877	-20.995	-12.868
1925	4.775	16.285	18.185	10.813
1926	-5.821	-17.163	-18.481	-8.528
1927	5.327	17.751	18.454	8.722
1928	-6.750	-16.024	-16.735	-7.687
1929	5.482	15.208	16.417	7.986
1930	-6.469	-16.324	-17.540	-6.634
1931	6.791	16.698	15.952	7.860
1932	-7.250	-19.349	-17.472	-3.203
1933	10.752	18.929	15.034	3.073
1934	-11.614	-19.778	-14.790	-.047
1935	13.824	20.834	14.171	.598
1936	-16.251	-21.040	-13.885	-.119
1937	17.506	21.280	13.374	-3.284
1938	-17.733	-21.626	-12.074	3.793
1939	17.658	21.139	10.920	-4.645

MEAN OF DIFFERENCED SERIES .6538D+00

MEAN SET EQUAL TO ZERO

VARIANCE OF Z SERIES = .1397D+03

VARIANCE OF DIFFERENCED SERIES = .2034D+03

AUTOCORRELATIONS OF STATIONARY SERIES

	.6928	-.0170	-.6925	-.9347	-.6119	.0651	.6788	.8663	.5296	-.1112	-.6601	-.7927
SE	.1155	.1617	.1617	.1973	.2495	.2687	.2689	.2909	.3235	.3348	.3353	.3522
	-.4505	.1489	.6363	.7247	.3814	-.1771	-.6106	-.6639	-.3205	.1996	.5815	.6053
SE	.3752	.3824	.3832	.3970	.4143	.4189	.4199	.4316	.4450	.4481	.4493	.4592
	.2691	-.2143	-.5508	-.5476	-.2195	.2252	.5185	.4957	.1814	-.2311	-.4911	-.4494
SE	.4697	.4717	.4730	.4815	.4898	.4911	.4924	.4997	.5062	.5070	.5085	.5147

PARTIAL AUTOCORRELATIONS

	.6928	-.9554	-.1695	-.0828	.0394	-.0153	-.0592	.0153	-.0416	.0294	.0486	.0071
SE	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155
	-.1005	.1264	-.0735	.0172	.0878	-.1181	.0403	.0056	-.0087	-.0198	-.0002	.0506
SE	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155
	-.0303	-.0143	.0773	-.0341	-.0157	.0269	-.0126	.0650	-.0249	-.0787	.0933	-.0200
SE	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155

```
MODEL FITTED

     NONSEASONAL     P= 2     D= 1     Q= 2
        SEASONAL    BP= 0    BD= 1    BQ= 0
     PERIODICITY    MQ=  4

INITIAL VALUES

 PHI:     .9000     .4500

  TH:    -.5806     .0737

 TRANSFORMED PARAMETERS:    .8820    .4500   -.6268    .0737

 MINIMUM BOUNDS        :   -.9800   -.9800   -.9800   -.9800
 MAXIMUM BOUNDS        :    .9800    .9800    .9800    .9800

 CONVERGED AFTER 17 ITERATIONS AND  76 FUTION VALUES    F = ,    .73252072E+02
        .697407E+00        -.100000E+01         .794877E+00        -.267574E+00

PARAMETERS FIXED
    2


FINAL VALUES OF PARAMETERS FROM SEARCH


         X(PHI)   =  .697407-1.000000

           SE     =  .002250*********

         X(THETA) =  .794877 -.267574

           SE     =  .055338  .115278
```

PARAMETER ESTIMATES

MEAN = .000000

SE = *******

CORRELATION MATRIX

1.000			
******	******		
.014	******	1.000	
.002	******	.016	1.000

ARIMA PARAMETERS (B-J SIGNS)

PHI	=	1.3948	-1.0000
SE	=	*******	*******
THETA	=	1.0076	-.2676
SE	=	.1145	.1153

ROOTS OF MA(Q) POLYNOMIAL

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
.5038	-.1174	.5173	-13.1154	-27.4486
.5038	.1174	.5173	13.1154	27.4486

ROOTS OF AR(P) POLYNOMIAL

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
.6974	-.7167	1.0000	-45.7807	-7.8636
.6974	.7167	1.0000	45.7807	7.8636

RESIDUALS

YEAR	1ST	2ND	3RD	4TH
1921		-.216	.796	-1.025
1922	.645	-.022	-.372	-.240
1923	-.935	-.988	1.455	-.159
1924	.112	1.791	.978	1.045
1925	2.520	-.984	-1.420	.566
1926	-1.768	-.164	-.055	.075
1927	-1.169	.595	-.066	.507
1928	.067	2.045	.908	-.001
1929	-.775	-.906	-.019	.519
1930	-.663	-.122	-1.186	.345
1931	-.831	-.338	-.665	1.728
1932	-.342	-2.184	.157	2.561
1933	.286	.332	-.359	.582
1934	-.184	-.846	.379	1.413
1935	.421	1.552	.386	1.640
1936	-1.365	.411	-.010	-1.912
1937	1.863	-.868	-.174	-.602
1938	-.338	-.355	.090	-.807
1939	-.543	-.029	-.791	.473

STUDENTIZED RESIDUAL OF 2.5054 AT T= 52 (4 1932)

```
TEST-STATISTICS ON RESIDUALS
----------------------------


          MEAN=   .3265D-01
      ST.DEV.=    .1132D+00
      OF MEAN
      T-VALUE=    .2885

     SKEWNESS=    .4235          ( SE =   .2828 )
     KURTOSIS=  3.1569           ( SE =   .5657 )

SUM OF SQUARES=   .7211D+02

 DURBIN-WATSON=  2.0681

STANDARD ERROR=   .1022D+01
OF RESID.
     VARIANCE=    .1045D+01
     OF RESID.
```

AUTOCORRELATIONS OF RESIDUAL

	-.0370	.0102	.0929	-.0263	-.1074	-.0442	-.1588	-.0051	-.0294	-.1387	-.0785	.0778
SE	.1155	.1156	.1156	.1166	.1167	.1180	.1182	.1211	.1211	.1211	.1232	.1239
	-.0076	.0783	-.0058	-.0571	-.0125	.1046	-.1142	-.0243	.0412	-.0611	-.1315	-.0591
SE	.1246	.1246	.1252	.1252	.1256	.1256	.1267	.1281	.1282	.1283	.1287	.1305
	.0262	-.0941	.0877	.0539	-.0043	.0504	-.0159	.0396	.0934	.0127	-.0074	.0227
SE	.1309	.1309	.1318	.1326	.1329	.1329	.1332	.1332	.1333	.1342	.1342	.1342

THE LJUNG-BOX Q VALUE IS 13.36 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (20)
THE PIERCE QS VALUE IS 1.39 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (6)

PARTIAL AUTOCORRELATIONS

	-.0370	.0089	.0938	-.0196	-.1124	-.0618	-.1592	.0019	-.0215	-.1321	-.1183	.0356
SE	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155
	.0036	.0608	-.0516	-.1163	-.0818	.0961	-.0689	-.0684	-.0162	-.0710	-.1450	-.1035
SE	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155
	.0267	-.1742	.0228	.0310	-.0590	-.0667	-.0829	-.0139	.0222	.0157	-.0421	-.0637
SE	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155	.1155

APPROXIMATE TEST OF RUNS ON AUTOCORRELATION FUNCTION

```
NUM.DATA=  36
 NUM.(+)=  18
 NUM.(-)=  18
 T-VALUE= -.33820
```

APPROXIMATE TEST OF RUNS ON RESIDUALS

```
NUM.DATA=  75
 NUM.(+)=  38
 NUM.(-)=  37
 T-VALUE=  .00000
```

AUTOCORRELATIONS OF SQUARED RESIDUAL

	-.0899	.2035	-.0456	.1470	-.0930	-.1142	-.0943	-.0767	-.1045	-.0579	-.1843	-.0144
SE	.1155	.1164	.1211	.1213	.1236	.1246	.1260	.1269	.1275	.1286	.1290	.1325
	.0192	.0351	-.1470	.1848	.0286	.1006	-.0287	-.0312	-.0672	-.0931	-.0843	-.1346
SE	.1325	.1325	.1326	.1348	.1381	.1382	.1392	.1393	.1394	.1398	.1406	.1413
	-.0200	-.0842	.1497	-.0958	.1382	.0337	.2087	-.0258	-.0490	.0918	.0674	-.1096
SE	.1430	.1430	.1437	.1457	.1466	.1483	.1484	.1523	.1523	.1525	.1533	.1537

THE LJUNG-BOX Q VALUE IS 24.38 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (20)
THE PIERCE QS VALUE IS 7.78 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED (6)

BACKWARD RESIDUALS

YEAR	1ST	2ND	3RD	4TH
1920	-.806	.754	-1.194	-.247
1921	-.005	.243	1.416	1.461
1922	-.201	2.005	1.243	-.346
1923	.168	-1.054	-2.580	.682
1924	.092	-1.287	1.290	-.491
1925	-.059	.256	1.507	.015
1926	1.123	.518	.492	-1.807
1927	-.747	-.299	.286	.209
1928	-.747	-1.030	.017	-.537
1929	.477	-.834	.709	.198
1930	.307	-1.630	1.341	2.690
1931	-.027	-1.683	.558	-.027
1932	.537	-.182	1.166	1.835
1933	.661	.053	.734	-.801
1934	-.133	-1.486	1.128	-1.089
1935	-.002	1.114	-2.497	.588
1936	-.625	-.071	-.394	-.388
1937	-.646	.353	.036	-.088
1938	.663	-.561	.127	

```
SECOND PART:
DERIVATION OF THE MODELS FOR THE COMPONENT

SERIES TITLE: SIMUL

MODEL PARAMETERS
(2,1,2)(0,1,0)

PARAMETER VALUES PASSED FROM ARIMA ESTIMATION (TRUE SIGNS)

THETA PARAMETERS
 1.00  -1.01    .27

BTHETA PARAMETERS
 1.00

PHI PARAMETERS
 1.00  -1.39   1.00

BPHI PARAMETERS
 1.00

NUMERATOR OF THE MODEL
 1.0000 -1.0076   .2676

STATIONARY AUTOREGRESSIVE TREND
 1.0000

NON-STATIONARY AUTOREGRESSIVE TREND
 1.0000 -2.0000  1.0000

AUTOREGRESSIVE TREND
 1.0000 -2.0000  1.0000

STATIONARY AUTOREGRESSIVE CYCLE
 1.0000
```

```
NON-STATIONARY AUTOREGRESSIVE CYCLE
 1.0000 -1.3948  1.0000

AUTOREGRESSIVE CYCLE
 1.0000 -1.3948  1.0000

STATIONARY AUTOREGRESSIVE SEASONAL COMPONENT
 1.0000

NON-STATIONARY AUTOREGRESSIVE SEASONAL COMPONENT
 1.0000  1.0000  1.0000  1.0000

AUTOREGRESSIVE SEASONAL COMPONENT
 1.0000  1.0000  1.0000  1.0000

STATIONARY AUTOREGRESSIVE SEASONALLY ADJUSTED COMPONENT
 1.0000

NON-STATIONARY AUTOREGRESSIVE SEASONALLY ADJUSTED COMPONENT
 1.0000 -3.3948  4.7896 -3.3948  1.0000

AUTOREGRESSIVE SEASONALLY ADJUSTED COMPONENT
 1.0000 -3.3948  4.7896 -3.3948  1.0000

TOTAL DENOMINATOR
 1.0000 -2.3948  2.3948 -1.0000 -1.0000  2.3948 -2.3948  1.0000
```

HARMONIC FUNCTIONS

$$\frac{F(X)}{H(X)} = \frac{F(X)}{T(X)\ C(X)\ S(X)}$$

F(X)

2.0868 -2.5543 .5351

T(X)

6.0000 -8.0000 2.0000

C(X)

3.9455 -5.5793 2.0000

S(X)

4.0000 6.0000 4.0000 2.0000

N(X), FORMED FROM THE PRODUCT T(S)C(X)

47.9900 -78.6188 42.2080 -13.5793 2.0000

H(X), FORMED FROM THE PRODUCT T(X)C(X)S(X) = N(X)S(X)

26.9405 -40.0990 9.5793 17.0495 -26.9405 21.0495 -9.5793 2.0000

$$\frac{F(X)}{H(X)} = QT(X) + \frac{RT(X)}{H(X)}$$

QT(X) QUOTIENT
.0000

RT(X) REMAINDER
2.0868 -2.5543 .5351 .0000 .0000 .0000 .0000 .0000

```
  RT(X)    = U(X) + V(X)
+ -------    ----   ----
  N(X)S(X)   N(X)   S(X)

U(X)
    2.4397    -3.6277     1.4277     -.2355

V(X)
     .4349      .6423      .2355


DUM(X) = RT(X) - U(X)S(X) - V(X)N(X). THIS SHOULD BE ZERO

      .0000       .0000       .0000       .0000       .0000       .0000       .0000


   U(X)    = UT(X) + UC(X)
+ -------    -----   -----
  T(X)C(X)   T(X)    C(X)

UT(X)
     .1872     -.1757

UC(X)
     .1619     -.0598


DUM(X) =  U(X) - UT(X)C(X) - UC(X)T(X). THIS SHOULD BE ZERO

      .0000       .0000       .0000       .0000
```

LOCAL MINIMA	FREQUENCY (RADIANS)	CONVERGENCE TEST
SEASONAL SPECTRUM.LOCAL MINIMA		
.082046	.0000	0
-.016577	2.3296	0
MINIMUM MINIMORUM		
-.016577		
CYCLE SPECTRUM. SIMPLE MINIMUM		
.019241	3.1416	0
TREND SPECTRUM. SIMPLE MINIMUM		
.022683	3.1416	0

```
                  MODELS FOR THE COMPONENTS
                  -------------------------

TREND NUMERATOR
    1.0000      .2986     -.7014
TREND DENOMINATOR
    1.0000    -2.0000     1.0000
INNOV. VAR.      .03234


SEAS. NUMERATOR
    1.0000     1.5377     1.2223      .1616
SEAS. DENOMINATOR
    1.0000     1.0000     1.0000     1.0000
INNOV. VAR.      .10261


CYCLE NUMERATOR
    1.0000      .6539     -.3461
CYCLE DENOMINATOR
    1.0000    -1.3948     1.0000
INNOV. VAR.      .05559


IRREGULAR
VAR.       .02535


SEASONALLY ADJUSTED NUMERATOR
    1.0000    -1.6594      .9053     -.0912     -.0462

SEASONALLY ADJUSTED DENOMINATOR
    1.0000    -3.3948     4.7896    -3.3948     1.0000
INNOV. VAR.      .35871
```

MA ROOTS OF TREND

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
.7014	.0000	.7014	.0000	-
-1.0000	.0000	1.0000	180.0000	2.0

TOTAL SQUARED ERROR= .6260090D-33

MA ROOTS OF SEAS.

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
-.1616	.0000	.1616	180.0000	2.0
-.6881	-.7256	1.0000	-133.4786	-2.6971
-.6881	.7256	1.0000	133.4777	2.6971

TOTAL SQUARED ERROR= .8306315D-21

MA ROOTS OF CYCLE

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
.3461	.0000	.3461	.0000	-
-1.0000	.0000	1.0000	180.0000	2.0

TOTAL SQUARED ERROR= .9265876D-33

MA ROOTS OF SEASONALLY ADJUSTED SERIES

REAL PART	IMAGINARY PART	MODULUS	ARGUMENT (DEG.)	PERIOD
.5844	.3099	.6615	27.9326	12.8881
.5844	-.3099	.6615	-27.9326	-12.8881
-.1619	.0000	.1619	180.0000	2.0
.6524	.0000	.6524	.0000	-

TOTAL SQUARED ERROR= .1058303D-27

AUTOCORRELATION FUNCTION OF COMPONENTS (STATIONARY TRANSFORMATION)

	TREND			ADJUSTED		
LAG	COMPONENT	ESTIMATOR	ESTIMATE	COMPONENT	ESTIMATOR	ESTIMATE
1	.056	.368	.327	-.707	-.685	-.657
2	-.444	.149	.179	.221	.085	-.015
3	.000	.406	.484	-.003	.368	.504
4	.000	.006	-.125	-.010	-.508	-.550
5	.000	.049	-.138	.000	.334	.224
6	.000	.149	.013	.000	-.084	.111
7	.000	-.187	-.408	.000	-.016	-.178
8	.000	-.232	-.332	.000	.009	.026
VAR.(*)	.051	.005	.005	1.644	.823	.802

(*) IN UNITS OF VAR(A)

AUTOCORRELATION FUNCTION OF COMPONENTS (STATIONARY TRANSFORMATION)

	IRREGULAR			SEASONAL		
LAG	COMPONENT	ESTIMATOR	ESTIMATE	COMPONENT	ESTIMATOR	ESTIMATE
1	.000	-.674	-.650	.740	.253	.266
2	.000	.080	-.001	.301	-.538	-.438
3	.000	.374	.502	.033	-.359	-.201
4	.000	-.497	-.548	.000	.345	.411
5	.000	.321	.230	.000	.246	.071
6	.000	-.084	.085	.000	-.172	-.411
7	.000	-.028	-.187	.000	-.315	-.363
8	.000	-.006	.025	.000	-.126	.031
VAR.(*)	.025	.004	.004	.501	.071	.059

(*) IN UNITS OF VAR(A)

	CYCLE		
LAG	COMPONENT	ESTIMATOR	ESTIMATE
1	.276	.509	.497
2	-.224	-.138	-.036
3	.000	-.436	-.254
4	.000	-.548	-.544
5	.000	-.229	-.330
6	.000	.124	.027
7	.000	.121	.004
8	.000	.045	.008
VAR.(*)	.086	.030	.024

(*) IN UNITS OF VAR(A)

CROSSCORRELATION FUNCTION

LAG	P(T)S(T-K) ESTIMATOR	P(T)S(T-K) ESTIMATE	S(T)U(T-K) ESTIMATOR	S(T)U(T-K) ESTIMATE	P(T)U(T-K) ESTIMATOR	P(T)U(T-K) ESTIMATE
-4	.164	.088	.134	.058	-.049	.009
-3	.151	.007	.029	.022	.256	.313
-2	-.224	-.319	-.045	-.024	-.299	-.405
-1	-.405	-.409	-.151	-.107	.104	.104
0	-.405	-.410	.119	.142	.206	.232
1	-.224	-.318	.071	.052	-.600	-.648
2	.151	.008	.071	.054	.206	.226
3	.164	.085	.119	.137	.104	.114
4	-.132	-.056	-.151	-.089	-.299	-.406

CROSSCORRELATION FUNCTION

LAG	S(T)C(T-K) ESTIMATOR	S(T)C(T-K) ESTIMATE	P(T)C(T-K) ESTIMATOR	P(T)C(T-K) ESTIMATE	C(T)U(T-K) ESTIMATOR	C(T)U(T-K) ESTIMATE
-4	-.278	-.253	-.045	-.075	-.096	-.063
-3	.214	.153	-.081	.015	.133	.184
-2	.398	.335	-.377	-.402	-.120	-.212
-1	.139	.182	-.240	-.387	.147	.161
0	-.085	.094	.161	.037	.172	.239
1	-.085	.094	-.240	-.387	-.314	-.332
2	.139	.180	-.377	-.401	.172	.236
3	.398	.332	-.081	.015	.147	.169
4	.214	.156	-.045	-.077	-.120	-.214

THIRD PART:
ERROR ANALYSIS

ACF (LAG)	FINAL ESTIMATION ERROR			REVISION IN CONCURRENT ESTIMATORS		
	TREND	ADJUSTED	CYCLE	TREND	ADJUSTED	CYCLE
1	.816	.433	.690	.669	.093	.747
2	.484	-.183	.160	.378	-.404	.249
3	.232	-.278	-.113	.151	-.240	-.116
4	.067	.046	-.133	-.006	.247	-.212
VAR.(*)	.105	.081	.136	.311	.122	.216

TOTAL ESTIMATION ERROR (CONCURRENT ESTIMATOR)

ACF (LAG)	TREND	ADJUSTED	CYCLE
1	.706	.229	.725
2	.405	-.316	.215
3	.171	-.255	-.115
4	.012	.166	-.182
VAR.(*)	.416	.203	.352

VARIANCE OF THE REVISION ERROR (*)

ADDITIONAL PERIODS	TREND	ADJUSTED	CYCLE
0	.3109	.1224	.2162
4	.0089	.0118	.0122
8	.0018	.0002	.0006
12	.0000	.0000	.0000
16	.0000	.0000	.0000
20	.0000	.0000	.0000

PERCENTAGE REDUCTION IN THE STANDARD ERROR OF THE REVISION AFTER ADDITIONAL YEARS
(COMPARISON WITH CONCURRENT ESTIMATORS)

AFTER 1 YEAR	83.0666	68.9677	76.2182
AFTER 2 YEAR	92.3193	95.7094	94.7392
AFTER 3 YEAR	99.2579	99.4103	99.6868
AFTER 4 YEAR	99.9732	99.9694	99.9956
AFTER 5 YEAR	99.9984	99.9994	99.9984

VARIANCE OF THE REVISION ERROR FOR THE SEASONAL COMPONENT (ONE YEAR AHEAD ADJUSTMENT)

PERIODS AHEAD	VARIANCE (*)
0	.1224
1	.1448
2	.1968
3	.1980
4	.3545

AVERAGE PERCENTAGE REDUCTION IN RMSE FROM CONCURRENT ADJUSTMENT 14.0083

(*) IN UNITS OF VAR(A)

DECOMPOSITION OF THE SERIES: RECENT ESTIMATES

PERIOD	SERIES	TREND			ADJUSTED		
		ESTIMATE	STANDARD ERROR		ESTIMATE	STANDARD ERROR	
			TOTAL	DUE TO REVISION		TOTAL	DUE TO REVISION
-8	965.3820	959.8085	.3338	.0438	971.6025	.2914	.0153
-7	968.9920	959.9401	.3373	.0652	962.8999	.2916	.0187
-6	959.5950	960.0474	.3414	.0838	952.3454	.2916	.0187
-5	939.5820	960.1698	.3418	.0856	946.7184	.2950	.0484
-4	942.7580	960.2454	.3447	.0965	948.9390	.3115	.1110
-3	964.0260	960.3139	.3603	.1425	957.9729	.3239	.1422
-2	975.7680	960.4086	.4107	.2432	968.4747	.3420	.1796
-1	966.6750	960.4937	.5110	.3893	973.9214	.3435	.1824
0	965.2060	960.6030	.6591	.5700	971.3900	.4611	.3576
STANDARD ERROR OF FINAL ESTIMATOR			.3309			.2910	

PERIOD	SEASONAL			CYCLE		
	ESTIMATE	STANDARD ERROR		ESTIMATE	STANDARD ERROR	
		TOTAL	DUE TO REVISION		TOTAL	DUE TO REVISION
-8	-6.2205	.2914	.0153	11.7943	.3775	.0250
-7	6.0921	.2916	.0187	2.9278	.3792	.0442
-6	7.2496	.2916	.0187	-7.6389	.3837	.0731
-5	-7.1364	.2950	.0484	-13.5090	.3926	.1108
-4	-6.1810	.3115	.1110	-11.2893	.3933	.1131
-3	6.0531	.3239	.1422	-2.3149	.4267	.2005
-2	7.2933	.3420	.1796	8.0303	.5271	.3687
-1	-7.2464	.3435	.1824	13.4644	.6021	.4698
0	-6.1840	.4611	.3576	10.7750	.6065	.4754
STANDARD ERROR OF FINAL ESTIMATOR		.2910			.3767	

DECOMPOSITION OF THE SERIES: FORECAST

PERIOD	SERIES FORECAST	S.E.	TREND FORECAST	TREND STANDARD ERROR TOTAL	TREND STANDARD ERROR DUE TO REVISION	ADJUSTED FORECAST	ADJUSTED STANDARD ERROR TOTAL	ADJUSTED STANDARD ERROR DUE TO REVISION
1	968.3864	1.0223	960.7249	.8653	.7995	962.3161	.9840	.9400
2	959.6712	1.7482	960.8362	1.1090	1.0585	952.2715	1.5403	1.5125
3	940.1319	2.1327	960.9474	1.3694	1.3288	947.4100	1.9442	1.9223
4	944.5494	2.2029	961.0587	1.6460	1.6124	950.7413	2.1665	2.1469
5	966.3867	2.3561	961.1699	1.9380	1.9095	960.3164	2.2767	2.2580
6	977.8079	2.5124	961.2812	2.2448	2.2203	970.4081	2.3647	2.3467
7	967.6982	2.6730	961.3924	2.5658	2.5444	974.9763	2.5222	2.5054
8	965.1319	2.8308	961.5037	2.9004	2.8815	971.3237	2.8545	2.8396

PERIOD	SEASONAL FORECAST	SEASONAL STANDARD ERROR TOTAL	SEASONAL STANDARD ERROR DUE TO REVISION	CYCLE FORECAST	CYCLE STANDARD ERROR TOTAL	CYCLE STANDARD ERROR DUE TO REVISION
1	6.0702	.4858	.3889	1.5912	.6692	.5531
2	7.3998	.5388	.4535	-8.5647	.8302	.7398
3	-7.2781	.5400	.4548	-13.5374	.9143	.8331
4	-6.1919	.6747	.6087	-10.3174	.9153	.8342
5	6.0702	.6883	.6237	-.8535	.9707	.8946
6	7.3998	.7267	.6658	9.1270	1.0890	1.0218
7	-7.2781	.7275	.6668	13.5839	1.1487	1.0852
8	-6.1919	.8324	.7799	9.8200	1.1488	1.0853

CONFIDENCE INTERVAL AROUND A SEASONAL COMPONENT OF 0

	FINAL ESTIMATOR		CONCURRENT ESTIMATOR	
95% CONFIDENCE INTERVALL	-.5704	.5704	-.9037	.9037
70% CONFIDENCE INTERVALL	-.3018	.3018	-.4781	.4781

SAMPLE MEANS

	COMPLETE PERIOD	LAST THREE YEARS
SERIES	953.8672	962.4692
TREND	953.6684	960.0155
ADJUSTED	953.8691	962.4649
SEASONAL	-.0019	.0043

STANDARD ERROR OF ALTERNATIVE MEASURES OF GROWTH
(IN POINTS OF ANNUALIZED PERCENT GROWTH)

1. MONTHLY GROWTH OF THE MONTHLY SERIES

	TREND	SEASONALLY ADJ. SERIES
CONCURRENT ESTIMATOR	1.146	1.932
1 - PERIOD REVISION	1.054	1.586
2 - PERIOD REVISION	.963	1.482
3 - PERIOD REVISION	.890	1.480
4 - PERIOD REVISION	.857	1.272
5 - PERIOD REVISION	.820	1.253
6 - PERIOD REVISION	.808	1.241
7 - PERIOD REVISION	.808	1.240
8 - PERIOD REVISION	.806	1.240
FINAL ESTIMATOR	.803	1.240

3. ACCUMULATED GROWTH OVER THE LAST QUARTER OF PREVIOUS YEAR

	CONCURRENT ESTIMATOR		FINAL ESTIMATOR	
	TREND	SEONALLY ADJ. SERIES	TREND	SEASONALLY ADJ. SERIES
QUARTER 1	1.146	1.932	.803	1.240
QUARTER 2	1.003	1.362	.673	.895
QUARTER 3	.860	.870	.547	.620
QUARTER 4	.737	.489	.452	.402

(CENTERED) ESTIMATOR OF THE PRESENT ANNUAL GROWTH

STANDARD ERROR	TREND	SEAS. ADJ. SERIES	ORIGINAL SERIES
CONCURRENT ESTIMATOR	.999	1.683	1.748
FINAL ESTI-MATOR	.452	.402	.000

FOURTH PART:
ESTIMATES OF THE COMPONENTS

ORIGINAL SERIES

YEAR	1ST	2ND	3RD	4TH
1920	969.94	965.98	942.68	930.78
1921	943.36	954.45	953.61	956.10
1922	968.18	963.24	940.37	928.27
1923	941.26	952.06	952.43	955.13
1924	966.17	961.10	940.47	930.31
1925	946.12	957.33	954.89	955.54
1926	965.53	959.58	938.66	930.78
1927	946.10	957.89	955.43	956.27
1928	964.84	960.61	941.41	934.56
1929	948.62	959.60	956.81	957.95
1930	965.54	960.19	939.87	934.37
1931	948.75	960.10	955.73	958.10
1932	965.22	957.23	935.38	934.55
1933	952.42	963.36	956.55	958.78
1934	965.05	956.20	934.60	936.79
1935	956.88	968.87	961.44	964.23
1936	968.06	959.01	937.70	940.37
1937	961.71	973.94	966.00	965.38
1938	968.99	959.60	939.58	942.76
1939	964.03	975.77	966.68	965.21

SEASONAL COMPONENT (LEVELS)

YEAR	1ST	2ND	3RD	4TH
1920	4.41	7.97	-4.12	-8.30
1921	4.43	7.93	-3.81	-8.31
1922	4.38	7.65	-3.64	-8.19
1923	4.22	7.14	-3.66	-8.04
1924	4.53	7.28	-3.75	-8.18
1925	4.83	7.58	-4.00	-8.16
1926	4.66	7.48	-4.22	-7.87
1927	4.29	7.47	-4.00	-7.47
1928	3.93	7.26	-3.88	-6.98
1929	4.00	7.03	-4.27	-6.79
1930	4.15	7.12	-4.75	-6.47
1931	4.53	6.93	-5.77	-6.17
1932	5.45	7.02	-6.73	-6.09
1933	6.06	7.16	-7.30	-6.10
1934	6.21	7.14	-7.64	-5.79
1935	6.25	7.25	-7.74	-5.64
1936	6.18	7.31	-7.57	-6.04
1937	6.16	7.35	-7.18	-6.22
1938	6.09	7.25	-7.14	-6.18
1939	6.05	7.29	-7.25	-6.18
1940	6.07	7.40	-7.28	-6.19
1941	6.07	7.40	-7.28	-6.19

CYCLICAL COMPONENT (LEVELS)

YEAR	1ST	2ND	3RD	4TH
1920	13.15	5.70	-5.23	-12.97
1921	-12.93	-5.10	5.87	13.33
1922	12.99	4.97	-6.09	-13.43
1923	-12.76	-4.61	6.25	12.98
1924	11.47	3.17	-6.83	-12.48
1925	-10.16	-1.69	7.66	12.45
1926	9.73	1.11	-8.10	-12.40
1927	-9.39	-.98	7.79	11.78
1928	8.79	.81	-7.53	-11.40
1929	-8.49	-.48	7.93	11.63
1930	8.36	.10	-8.30	-11.85
1931	-8.16	.72	9.19	12.00
1932	7.34	-2.09	-10.23	-11.92
1933	-6.19	3.44	11.01	11.86
1934	5.37	-4.68	-12.09	-12.22
1935	-4.85	5.53	12.78	12.48
1936	4.43	-6.33	-13.25	-12.22
1937	-3.63	7.17	13.56	11.79
1938	2.93	-7.64	-13.51	-11.29
1939	-2.31	8.03	13.46	10.78

TREND

YEAR	1ST	2ND	3RD	4TH
1920	952.40	952.26	952.11	952.00
1921	951.85	951.67	951.45	951.13
1922	950.83	950.53	950.17	949.91
1923	949.74	949.66	949.84	950.09
1924	950.30	950.63	950.93	951.12
1925	951.33	951.39	951.29	951.22
1926	951.13	951.02	950.99	951.06
1927	951.21	951.41	951.66	951.92
1928	952.19	952.51	952.79	952.96
1929	953.06	953.09	953.12	953.11
1930	953.04	952.98	952.87	952.67
1931	952.48	952.40	952.33	952.32
1932	952.35	952.34	952.38	952.49
1933	952.59	952.71	952.85	953.07
1934	953.41	953.81	954.29	954.85
1935	955.46	956.01	956.57	957.16
1936	957.59	958.01	958.43	958.75
1937	959.11	959.42	959.63	959.81
1938	959.94	960.05	960.17	960.25
1939	960.31	960.41	960.49	960.60

SEASONALLY ADJUSTED SERIES

YEAR	1ST	2ND	3RD	4TH
1920	965.53	958.00	946.81	939.08
1921	938.93	946.52	957.42	964.41
1922	963.80	955.60	944.01	936.46
1923	937.04	944.92	956.09	963.17
1924	961.65	953.82	944.22	938.49
1925	941.29	949.75	958.89	963.70
1926	960.87	952.10	942.88	938.65
1927	941.81	950.43	959.43	963.74
1928	960.91	953.35	945.29	941.54
1929	944.62	952.56	961.08	964.74
1930	961.38	953.08	944.61	940.84
1931	944.22	953.17	961.50	964.27
1932	959.78	950.21	942.11	940.64
1933	946.36	956.19	963.84	964.88
1934	958.84	949.07	942.24	942.58
1935	950.63	961.61	969.18	969.86
1936	961.89	951.70	945.27	946.41
1937	955.55	966.59	973.18	971.60
1938	962.90	952.35	946.72	948.94
1939	957.97	968.47	973.92	971.39

IRREGULAR COMPONENT

YEAR	1ST	2ND	3RD	4TH
1920	-.02	.05	-.07	.05
1921	.01	-.05	.10	-.05
1922	-.02	.09	-.07	-.02
1923	.06	-.14	.00	.10
1924	-.12	.01	.13	-.15
1925	.12	.04	-.06	.03
1926	.02	-.03	-.02	-.01
1927	-.01	.00	-.02	.04
1928	-.07	.03	.03	-.02
1929	.05	-.05	.03	.00
1930	-.02	.00	.04	.02
1931	-.09	.06	-.02	-.05
1932	.09	-.04	-.04	.06
1933	-.04	.04	-.01	-.05
1934	.06	-.07	.04	-.05
1935	.03	.07	-.17	.23
1936	-.13	.01	.09	-.12
1937	.07	.00	-.01	.00
1938	.03	-.06	.06	-.02
1939	-.03	.04	-.04	.01

* *

* PROCESSING COMPLETED* *

* *

SIMUL: ORIGINAL SERIES

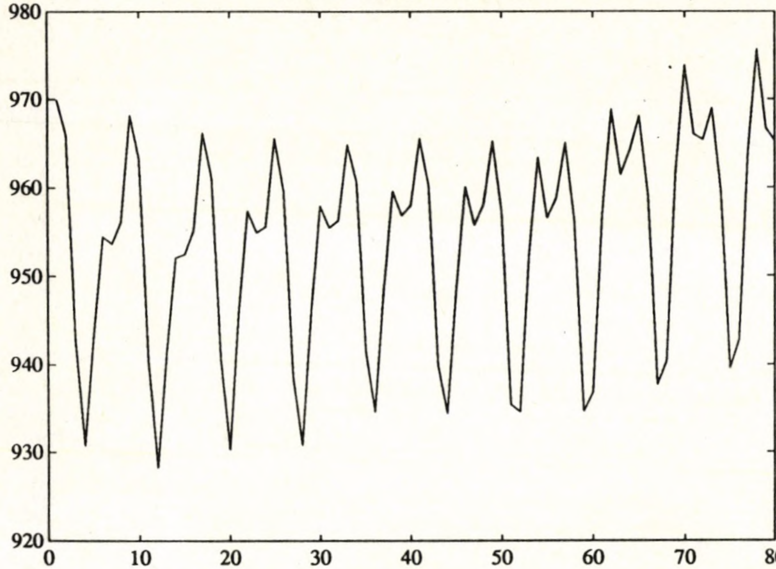


Fig. 1

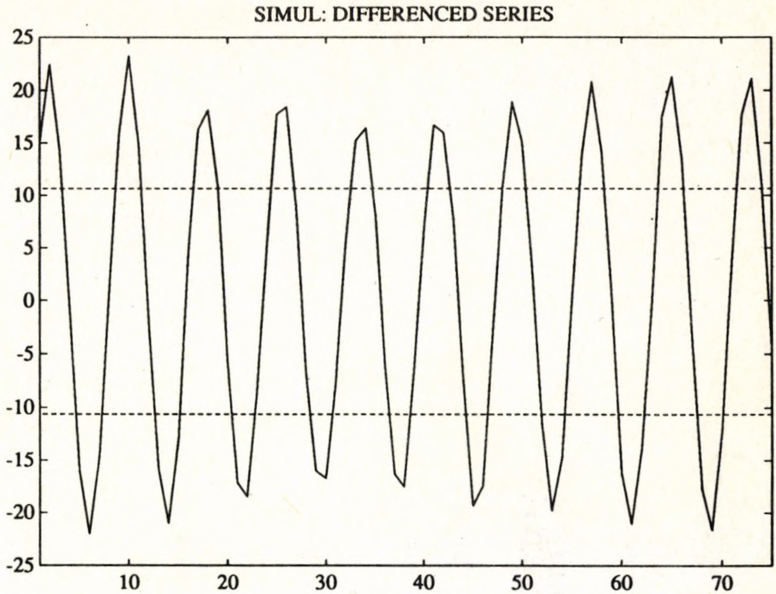

SIMUL: DIFFERENCED SERIES
25
20
15
10
5
0
-5
-10
-15
-20
-25
10
20
30
40
50
60
70

Fig. 2

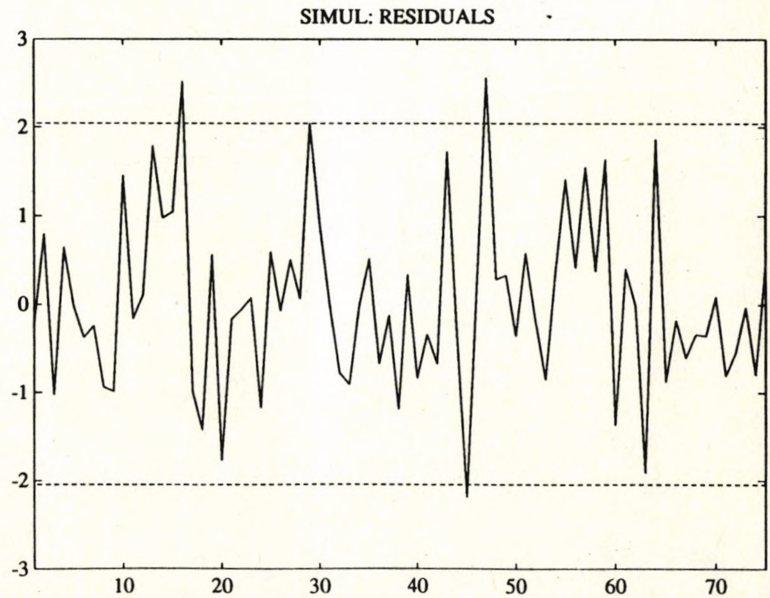
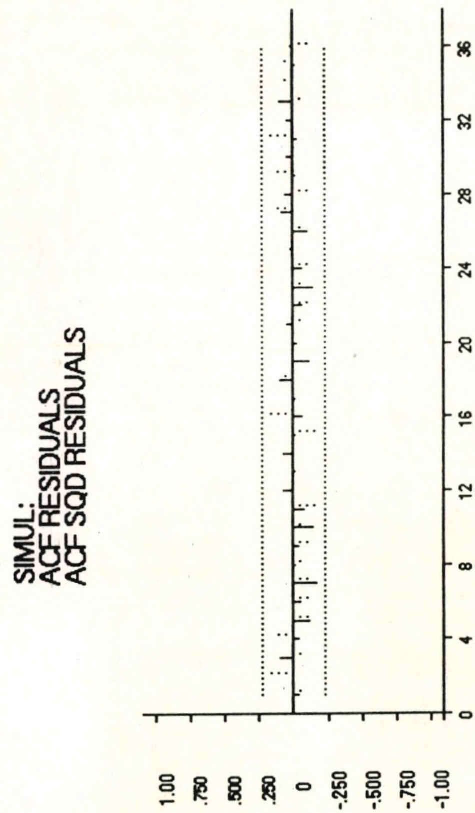

SIMUL: RESIDUALS
3
2
1
0
-1
-2
-3
10
20
30
40
50
60
70

Fig. 3


SIMUL:
ACF RESIDUALS
ACF SQD RESIDUALS
1.00
.750
.500
.250
0
-.250
-.500
-.750
-1.00
0
4
8
12
16
20
24
28
32
36

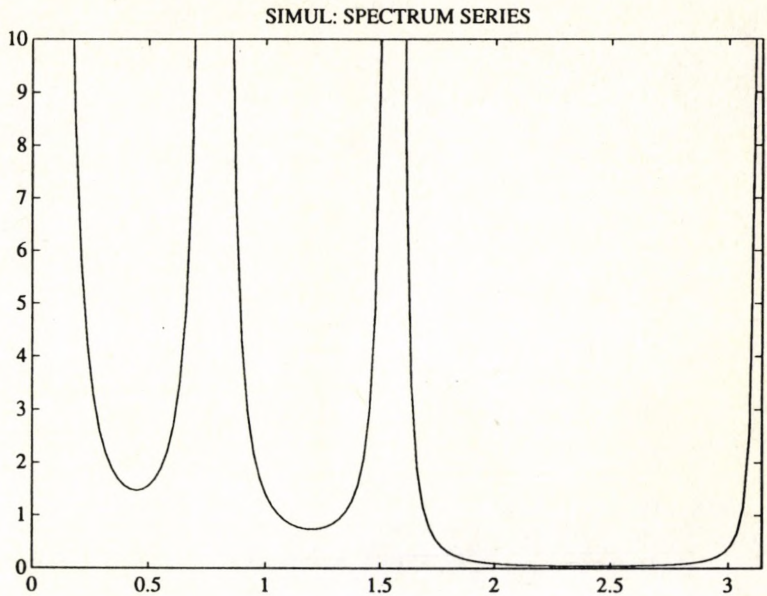

SIMUL: SPECTRUM SERIES
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3

Fig. 4

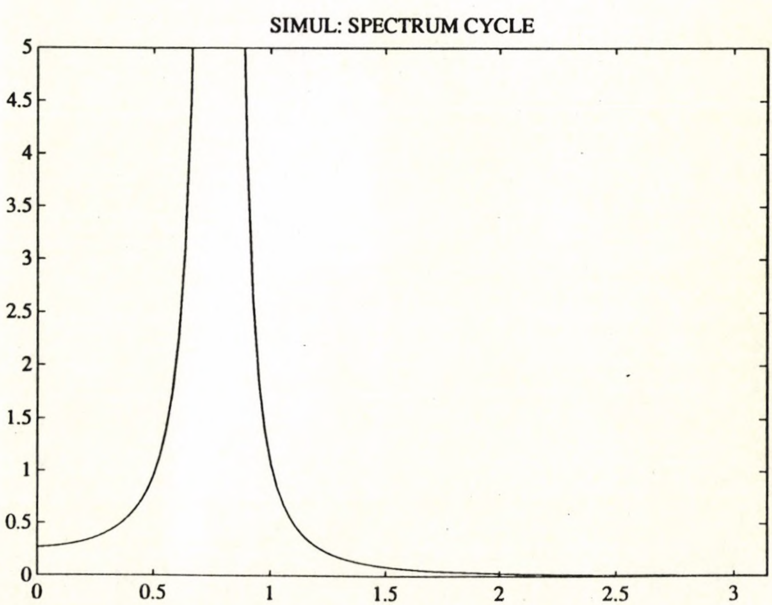

SIMUL: SPECTRUM CYCLE
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3

Fig. 5

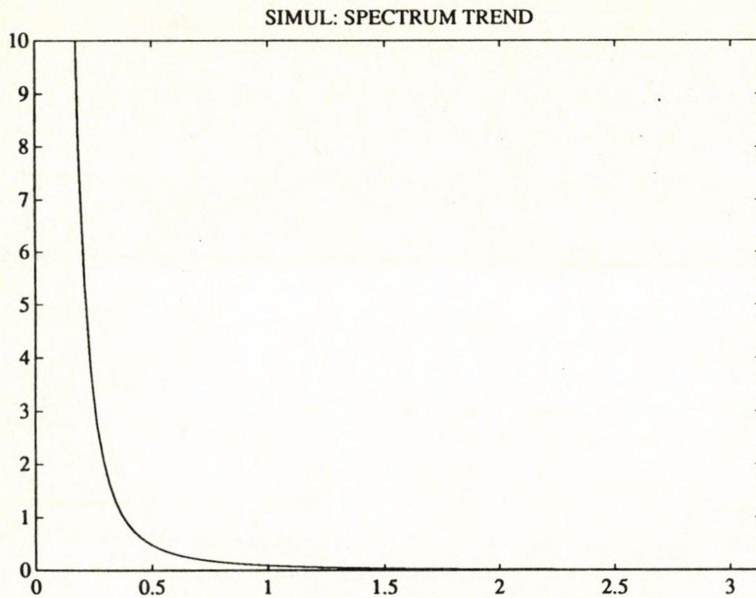

SIMUL: SPECTRUM TREND
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3

Fig. 6

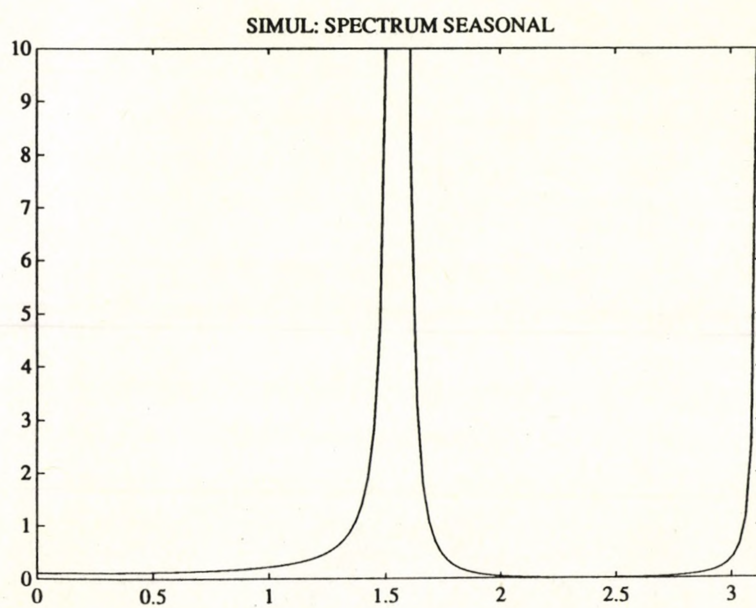
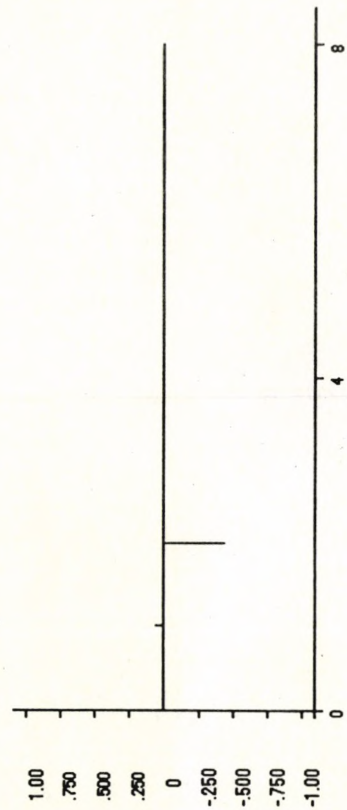
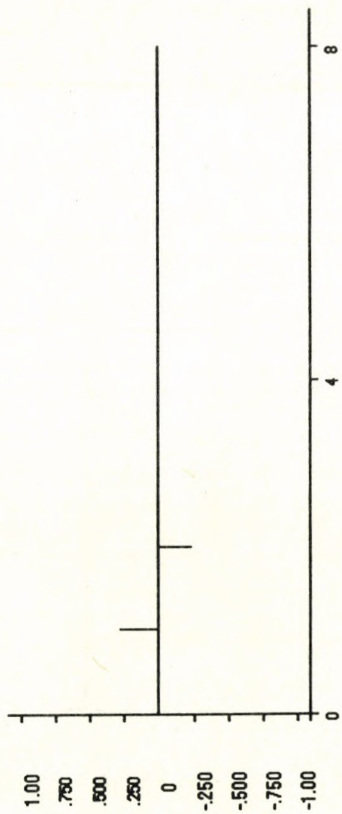

SIMUL: SPECTRUM SEASONAL
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3

Fig. 7

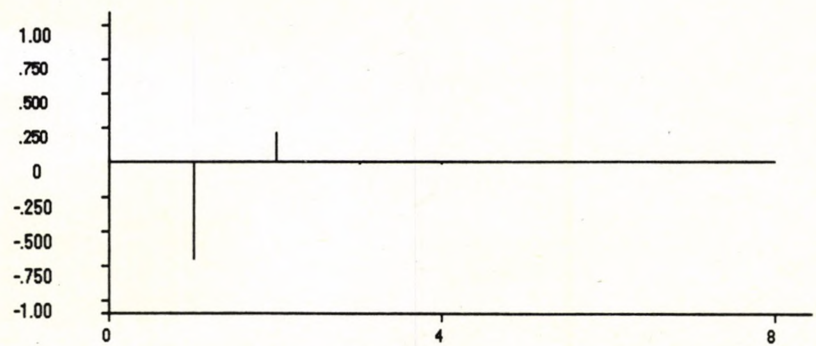
SIMUL: ACF OF TREND THEOR. COMPONENT

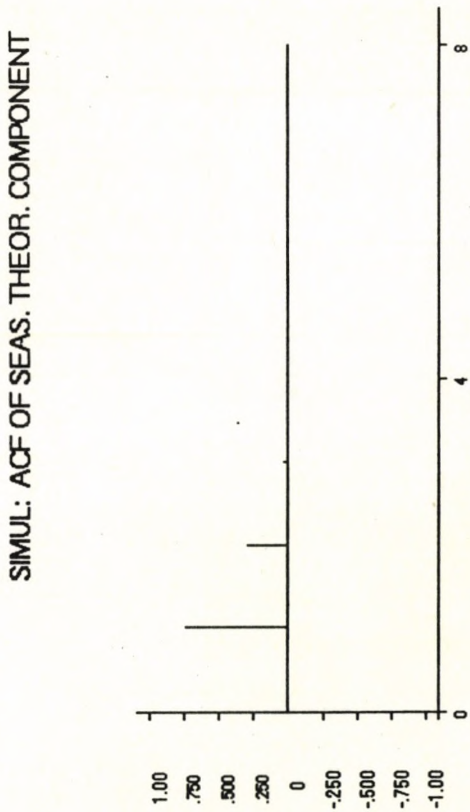


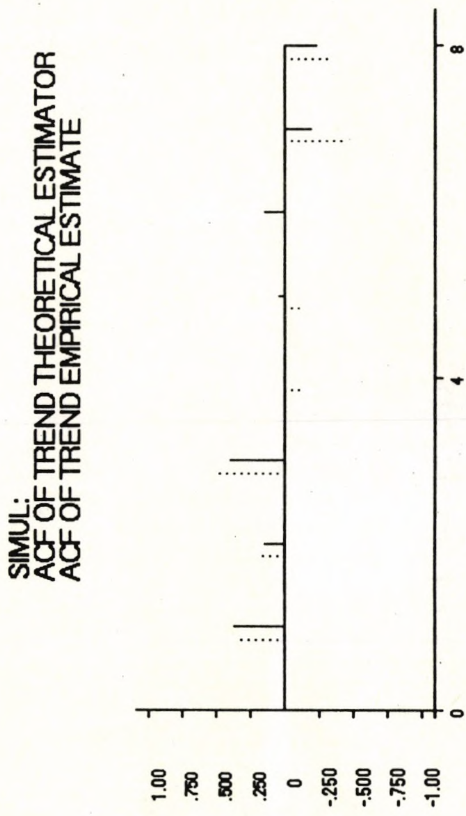
SIMUL: ACF OF CYCLE THEOR. COMPONENT



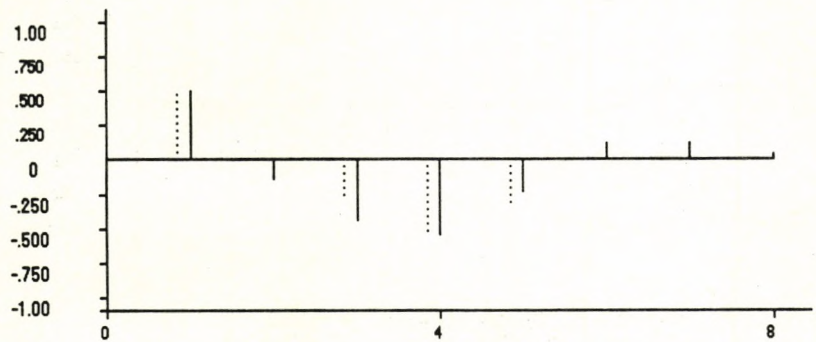
SIMUL: ACF OF S. ADJ. THEOR. COMPONENT

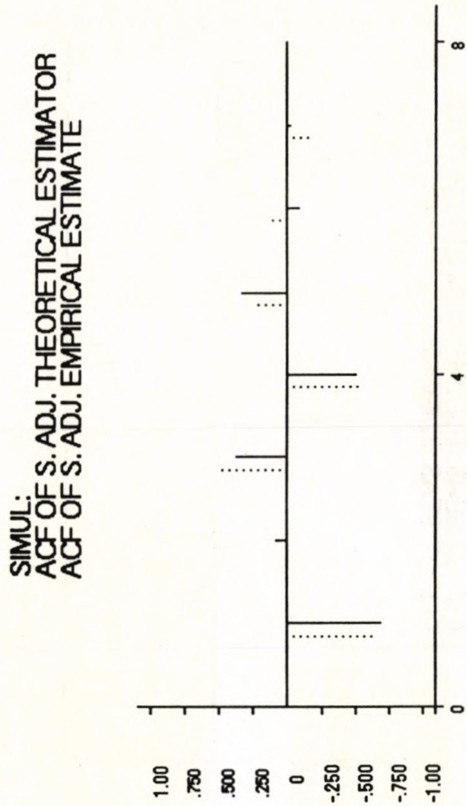


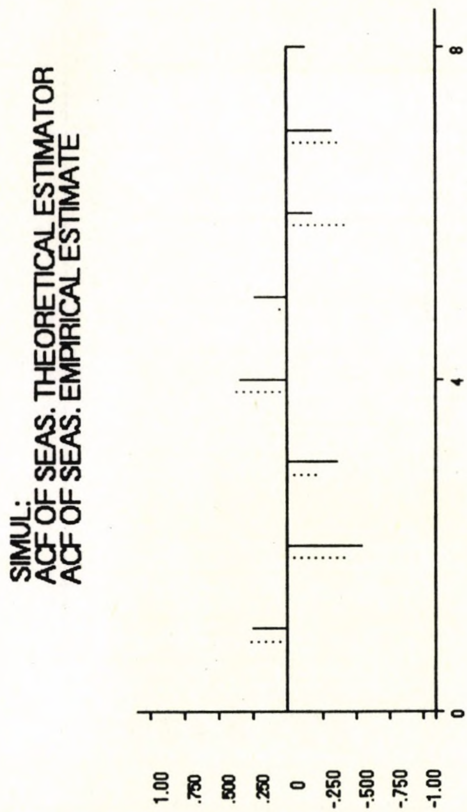

SIMUL: ACF OF SEAS. THEOR. COMPONENT
1.00
.750
.500
.250
0
-.250
-.500
-.750
-1.00
0
4
8


SIMUL:
ACF OF TREND THEORETICAL ESTIMATOR
ACF OF TREND EMPIRICAL ESTIMATE
1.00
.750
.500
.250
0
-.250
-.500
-.750
-1.00
0
4
8

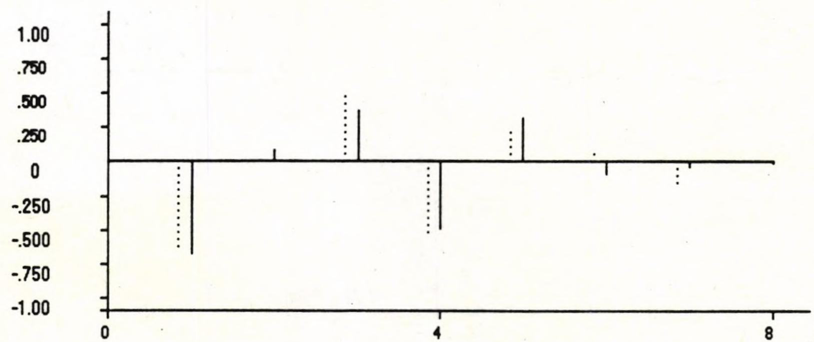
SIMUL:
ACF OF CYCLE THEORETICAL ESTIMATOR
ACF OF CYCLE EMPIRICAL ESTIMATE



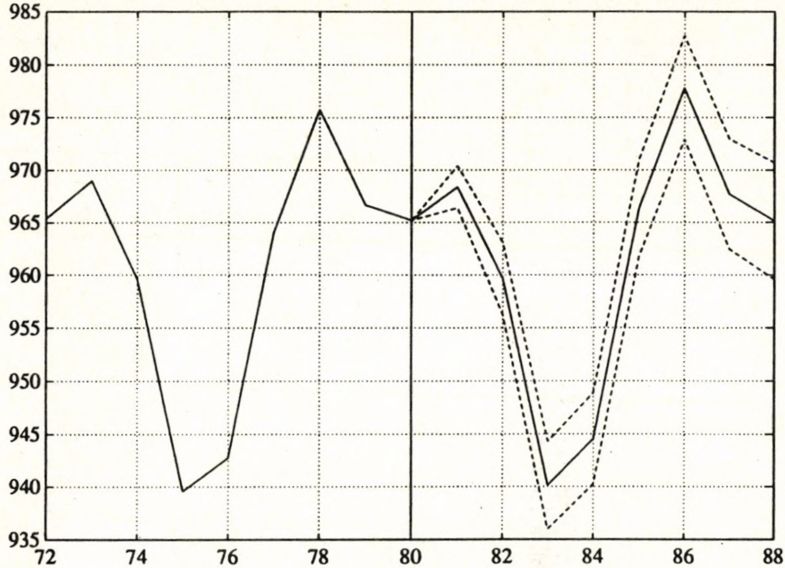

SIMUL:
ACF OF S. ADJ. THEORETICAL ESTIMATOR
ACF OF S. ADJ. EMPIRICAL ESTIMATE
1.00
.750
.500
.250
0
-.250
-.500
-.750
-1.00
0
4
8


SIMUL:
ACF OF SEAS. THEORETICAL ESTIMATOR
ACF OF SEAS. EMPIRICAL ESTIMATE
1.00
.750
.500
.250
0
-.250
-.500
-.750
-1.00
0
4
8

SIMUL:
ACF OF IRR. THEORETICAL ESTIMATOR
ACF OF IRR. EMPIRICAL ESTIMATE

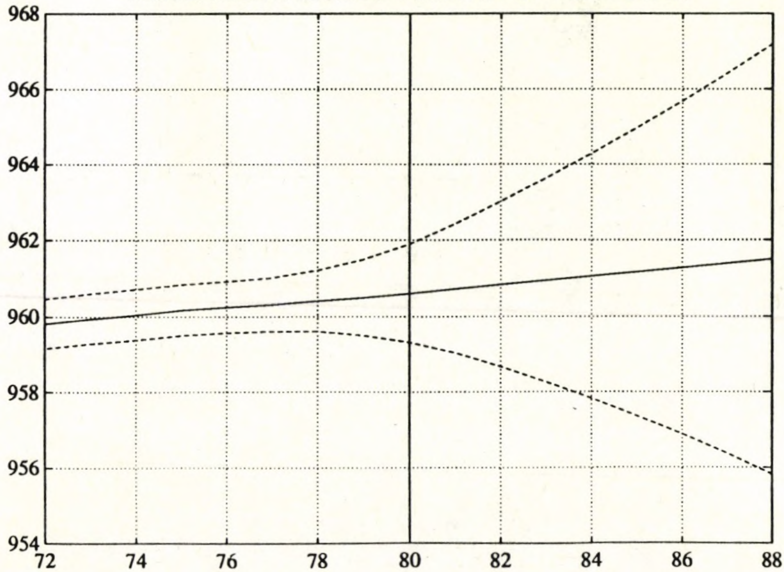


SIMUL: RECENT OBSERV. AND FORECAST OF AGGREGATE SERIES



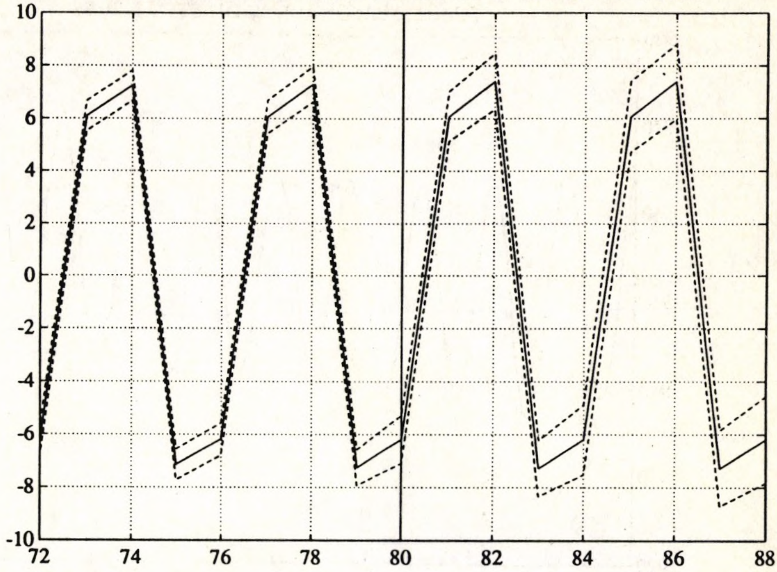
====: 95% C.I. Fig. 12

SIMUL: RECENT EST. AND FORECAST OF TREND COMP.



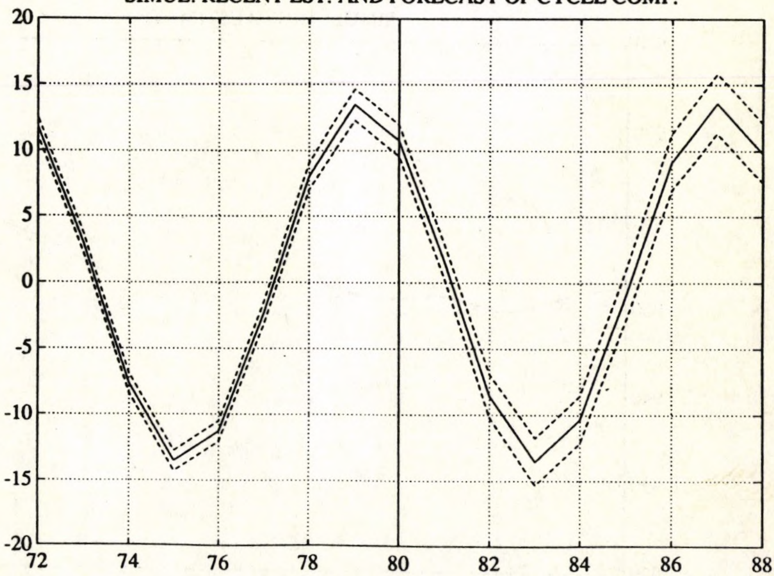
====: 95% C.I. Fig. 13

SIMUL: RECENT EST. AND FORECAST OF SEASONAL COMP.



====: 95% C.I. Fig. 14

SIMUL: RECENT EST. AND FORECAST OF CYCLE COMP.



====: 95% C.I. Fig. 15

SIMUL: SEASONALLY ADJUSTED SERIES

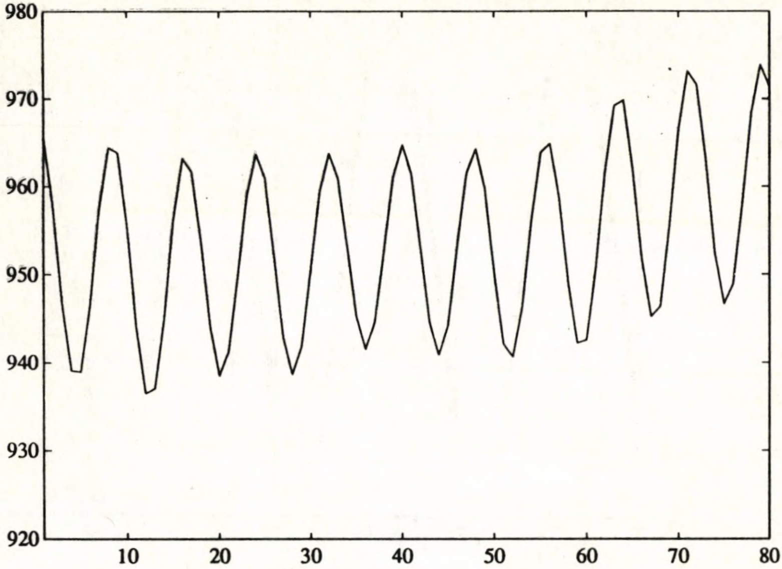


Fig. 8

SIMUL: TREND ESTIMATE

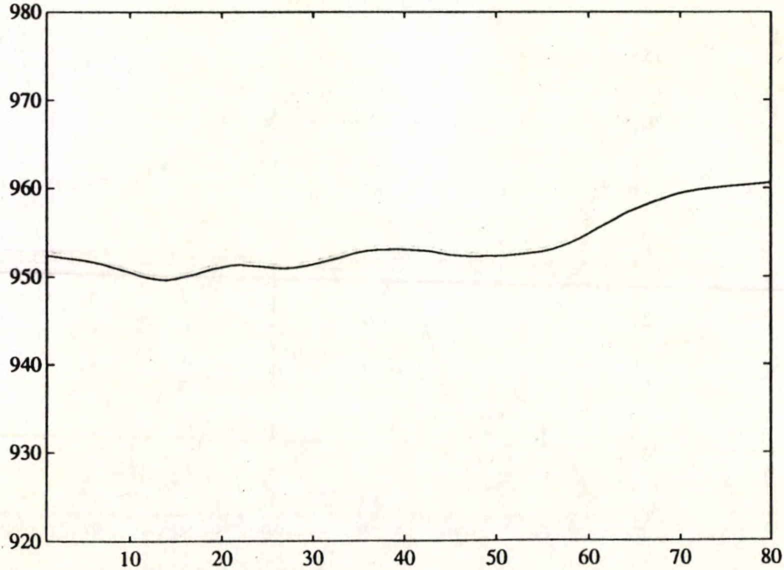


Fig. 9

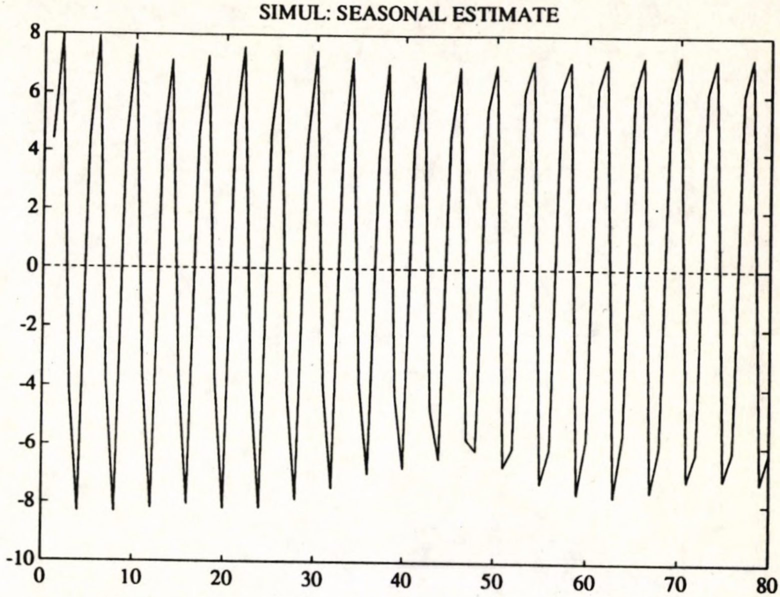

SIMUL: SEASONAL ESTIMATE
8
6
4
2
0
-2
-4
-6
-8
-10
0
10
20
30
40
50
60
70
80

Fig. 10

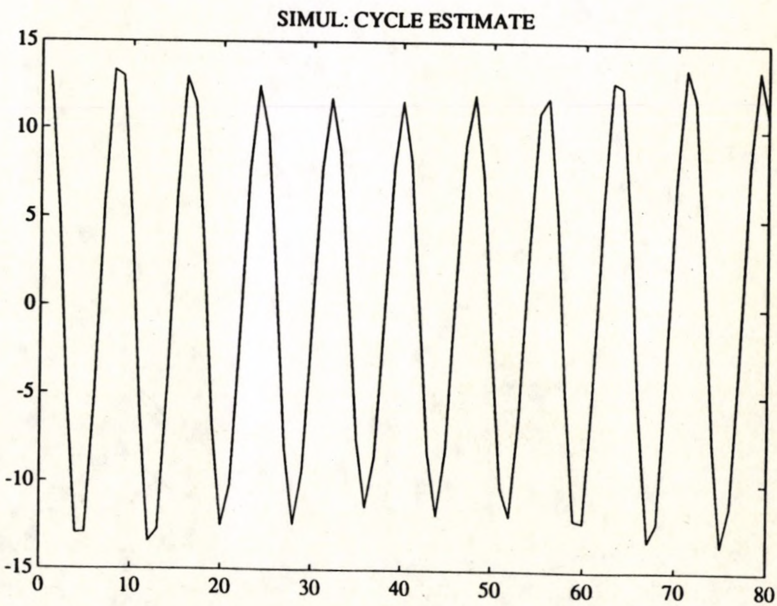

SIMUL: CYCLE ESTIMATE
15
10
5
0
-5
-10
-15
0
10
20
30
40
50
60
70
80

Fig. 11

PROGRAM SEATS
(Signal Extraction in Arima Time Series)

7. **Background Paper**

Notes on Unobserved Components Estimation with ARIMA Models

Time series analysts (often "econometricians") working in institutions involved with economic policy making or short-term economic analysis face two important professional demands: forecasting and unobserved components estimation (including sesonal adjustment, trend or cycle estimation and noise extraction.) Estimation of unobserved components is overwhelmingly done in practice by using "ad hoc" filters, permitting routine treatment of a large number of series. The most popular example is estimation of the seasonally adjusted series with the X11 or X11 ARIMA program.

Concerning forecasting, the decade of the seventies witnessed the proliferation of ARIMA models, which seemed to capture well the evolution of many series. Since this evolution is related to the presence of trend, seasonal and noise variation, the possibility of using ARIMA models in the context of unobserved components was soon recognized. Since the early work of Grether and Nerlove (1970) on stationary series, several approaches have been suggested. I shall concentrate on one which is becoming, in my opinion, a relatively powerful statistical tool in applied time series work (starting references are Cleveland and Tiao, 1976, and Box, Hillmer and Tiao, 1978.) I shall present an overview of the approach, addressing the issues of model specification, estimation of the components, diagnostic checking of the results and inference drawing.

1. MODEL SPECIFICATION

1.1. <u>General Framework</u>

Let an observed series, z_t, be the sum of several independent components, one of which is white noise. Hence

$$z_t = \Sigma_i \, z_{it} + u_t \, , \qquad (1.1)$$

where z_{it} denotes an unobservable component and u_t~niid $(0, \sigma_u^2)$. Particular cases of (1.1) are the trend (or trend/cycle) -seasonal-irregular and the signal plus noise decompositions of a time series.

The components are assumed to follow models of the type

$$z_{it} = \Psi_i(B)\, a_{it} \tag{1.2}$$

where $\Psi_i(B)$ represents a rational function in the backward shift operator B which can be expressed as

$$\Psi_i(B) = \Theta_i(B)/\Phi_i(B)\ , \tag{1.3}$$

where $\Theta_i(B)$ and $\Phi_i(B)$ are polynomials in B of finite order. The a_{it}'s are independent white noises with variance σ_i^2. Expressions (1.1), (1.2) and (1.3) imply that the observed series z_t also follows a linear model, say

$$z_t = \Psi(B)\, a_t\ , \tag{1.4}$$

where a_t is white noise and $\Psi(B)$ can also be expressed as the ratio of two polynomials in B of finite order,

$$\Psi(B) = \Theta(B)/\Phi(B)\ . \tag{1.5}$$

In other words, I assume the components, and hence the observed series, to follow ARIMA models. Further assumptions are the following:

a) The autoregressive polynomials $\Phi_i(B)$ share no root in common.

b) The zeroes of $\Phi(B)$, $\Phi_i(B)$ and $\Theta_i(B)$ lie on or outside the unit circle. Typically, these polynomials will contain unit roots and, in particular, the possibility of handling unit autoregressive roots has contributed decisively to the applied interest of the methodology.

c) The roots of $\theta(B)$ lie outside the unit circle, and hence the overall process (1.4) is invertible. (This is a sensible restriction, since noninvertibility of z_t would imply $\sigma_u^2 = 0$ and a zero in the spectrum of each component for the same frequency.)

From (1.1)-(1.3), z_t can be expressed as

$$z_t = \Sigma_i \; [\theta_i(B)/\phi_i(B)] \; a_{it} + u_t \; .$$

Removing the denominators and considering (1.4) and (1.5), the following two relationships are obtained

$$\phi(B) = \pi_i \; \phi_i \; (B) \; , \tag{1.6}$$

$$\theta(B) \; a_t = \Sigma_i \; \theta_i(B) \; \phi_i^*(B) \; a_{it} + \phi(B) u_t \; , \tag{1.7}$$

where

$$\phi_i^*(B) = \phi(B)/\phi_i(B) = \pi_{j(j \neq i)} \; \phi_j(B) \tag{1.8}$$

1.2. The Models for the Components

The components have been assumed to follow ARIMA models. How can these models be determined?

One approach is to specify a priori models that attempt to capture the essential properties ordinarily associated with a trend, a seasonal component and so on. This has been termed the "structural approach" and examples can be found in Engle (1978), Harvey and Todd (1983), and Hausman and Watson (1985).

Since observations are only available on z_t, an alternative approach is to start by identifying (specifying) the ARIMA model for z_t, and then deriving models for the components that are consistent with the overall "observable" model. Since in this case the structure is derived from the reduced form, the approach has been called the "reduced form" approach. Important references are Burman (1980), Hillmer and Tiao (1982), and Bell and Hillmer (1984).

Although we shall come back to the relationship between the two approaches, the second one will be the one followed in this paper. Thus we are interested in analysing what type of components can be extracted from ARIMA models. Given that there is some disagreement and confusion on that issue, it will prove helpful to look first at a rather simple example.

Consider a quarterly seasonal series that follows the model

$$\nabla_4 z_t = a_t \ , \tag{1.9}$$

where $\nabla_4 = 1-B^4$. The pseudospectrum of z_t, equal to

$$g_z(\omega) = \frac{1}{2(1-\cos 4\omega)} \sigma_a^2, \quad (0 \leq \omega \leq \pi), \tag{1.10}$$

is displayed in Figure 1 (the prefix "pseudo" will be removed in later references). It is symmetric around $\omega=\pi/2$ and presents three peaks, associated with the frequencies $\omega=0$, $\omega=\pi/2$ and $\omega=\pi$. One may try to decompose the series into orthogonal components, each one capturing a spectral peak. Thus we seek to express z_t as:

$$z_t = z_{1t} + z_{2t} + z_{3t} + u_t \quad , \tag{1.11}$$

where u_t is a white-noise residual.

The roots of the autoregressive (AR) polynomial $(1-B^4)$ are

$$\nabla_4 = (1-B)(1+B)(1+B^2) . \tag{1.12}$$

and since, in general, an AR factor of the type $(1-\Phi\ B^j)$ induces the factor $(1+\Phi^2-2\Phi \cos j\ \omega)$ in the denominator of the spectrum, it is easily seen that the spectral peak for $\omega=0$ is induced by the AR factor $(1-B)$ in (1.12) and, similarly, the peaks for $\omega=\pi$ and $\omega=\pi/2$ are induced by the AR factors $(1+B)$ and $(1+B^2)$, respectively. Thus, from (1.9), (1.11) and (1.12), in order to capture the individual spectral peaks, the z_i-components will be of the type:

$$\begin{aligned} (1-B)\ z_{1t} &= \alpha_1(B)\ a_{1t} \\ (1+B)\ z_{2t} &= \alpha_2(B)\ a_{2t} \\ (1+B^2)\ z_{3t} &= \alpha_3(B)\ a_{3t} \end{aligned} \tag{1.13}$$

where $\alpha_i(B)$ represents some polynomial in B, and a_{1t}, a_{2t} and a_{3t} are mutually independent white noises, also independent of u_t.

From (1.11) and (1.13), z_t can be expressed as

$$z_t = \frac{\alpha_1(B)}{1-B}\ a_{1t} + \frac{\alpha_2(B)}{1+B}\ a_{2t} + \frac{\alpha_3(B)}{1+B^2}\ a_{3t} + u_t,$$

and, removing the denominators, this expression will be compatible with (1.9) when the following identity (equivalent to (1.7)) holds:

$$a_t = (1+B)(1+B^2)\ \alpha_1(B)\ a_{1t} + (1-B)(1+B^2)\ \alpha_2(B)\ a_{2t} +$$

$$+ (1-B)(1+B)\ \alpha_3(B)\ a_{3t} + \nabla_4\ u_t. \tag{1.14}$$

Since the r.h.s. of (1.14) has to be white noise, the lag-4 autocorrelation of the term $\nabla_4\ u_t$ should cancel out with that of the other terms. Hence $\alpha_1(B)$ and/or $\alpha_2(B)$ have to be at least of order one, and/or $\alpha_3(B)$ has to be at least of order two. Therefore, we can assume that

$$\begin{aligned}\alpha_1(B) &= 1-\alpha_{11}\ B\\ \alpha_2(B) &= 1-\alpha_{21}\ B\\ \alpha_3(B) &= 1-\alpha_{31}\ B-\alpha_{32}\ B^2\ .\end{aligned} \tag{1.15}$$

Considering (1.13) and (1.15), the models for the four components are seen to depend on eight parameters: the four α-parameters and the four variances (σ_i^2 of a_{it}, i=1,2,3, and σ_u^2). These parameters have to satisfy the constraints implied by equating the autocovariances of the l.h.s. and the r.h.s. of (1.14). The variance and first four autocovariances yield the system of equations:

$$\sigma_a^2 = [1+3(1-\alpha_{11})^2+\alpha_{11}^2]\ \sigma_1^2 + [1+3(1+\alpha_{21})^2+\alpha_{21}^2]\ \sigma_2^2 +$$

$$+\ 2(1+\alpha_{31}^2+\alpha_{32}^2+\alpha_{32})\sigma_3^2 + 2\sigma_u^2$$

$$0 = 3(1-\alpha_{11})^2\ \sigma_1^2 - 3(1+\alpha_{21})^2\ \sigma_2^2 - \alpha_{31}(1-\alpha_{32})^2\ \sigma_3^2$$

$$0 = 2(1-\alpha_{11})^2\ \sigma_1^2 + 2(1+\alpha_{21})^2\ \sigma_2^2 - [(1+\alpha_{32})^2+\alpha_{31}^2]\sigma_3^2$$

$$0 = (1-\alpha_{11})^2\ \sigma_1^2 - (1+\alpha_{21})^2\ \sigma_2^2 + \alpha_{31}(1-\alpha_{32})\ \sigma_3^2$$

$$0 = -\alpha_{11}\ \sigma_1^2 + \alpha_{21}\ \sigma_2^2 + \alpha_{32}\ \sigma_3^2 - \sigma_u^2\ ,$$

and for other lags, the autocovariances in both sides of (1.14) are zero.

This provides a system of five equations with eight unknowns, and hence there will be an infinite number of parameter values in the component models which satisfy the system. The identification problem we face is similar to the one that appears in standard econometric models.

The model for the observed series is the reduced form, while the models for the components represent the associated structural form. For a particular reduced form, there are an infinite number of structures from which it can be generated. In order to select one, additional information has to be incorporated. The traditional approach in econometrics has been to set a priori some parameters in the structural model equal to zero (see Fisher, 1966). These zero-parameter restrictions were rationalized as reflecting a priori economic theory information, such as for example that some variables that affect demand of a commodity do not affect supply, and vice versa. In the case of our unobserved-components model, such a priori information is not available. We follow instead an alternative approach, originally suggested by Box, Hillmer and Tiao (1978) and Pierce (1978). The additional information will be the requirement that separable white noise should not be a part of either the trend or the seasonal component, and should go to the irregular. The variance of the irregular is thus maximized and the resulting decomposition has been termed "canonical" by Hillmer and Tiao (1982).

As a consequence, the following requirement will be imposed: Let z_{it} denote any component (not u_t). Then, z_{it} should not accept a decomposition of the type:

$$z_{it} = z^*_{it} + n_t \quad ,$$

where z^*_{it} and n_t are independent and the latter is white noise. (If such a decomposition were feasible, then the component z_{it} should be replaced with z^*_{it}, and n_t should be added to u_t.) We refer to this requirement as the "canonical" requirement.

Let $g_i(\omega)$ be the spectrum of z_{it} for $0 \leq \omega \leq \pi$. The canonical requirement implies that, for some ω in that range, $g_1(\omega)$ should be zero. From (1.13) and (1.15) it is found that $g_1(\omega)$ is monotonically decreasing in ω, hence no noise will contaminate

z_{1t} when $g_1(\pi)=0$. Since this condition implies the presence of the factor $(1+\cos\omega)$ in the numerator of $g_1(\omega)$, in the time domain it is equivalent to the presence of the factor $(1+B)$ in $\alpha_1(B)$ and, therefore, the model for z_{1t} is given by

$$(1-B)z_{1t} = (1+B)a_{1t}. \tag{1.16a}$$

Similarly, from (1.13) and (1.15), it is seen that $g_2(\omega)$ is monotonically increasing in the range $0\leq\omega\leq\pi$. Hence the canonical requirement implies $g_2(0)=0$, or equivalently the presence of the factor $(1-\cos\omega)$ in the numerator of $g_2(\omega)$. Thus, $\alpha_{21}=1$ and the model for z_{2t} is, therefore, equal to

$$(1+B)z_{2t} = (1-B)a_{2t} \quad . \tag{1.16b}$$

Concerning the third component z_{3t}, because of symmetry, its spectrum will reach a minimum of zero for $\omega=0$ and $\omega=\pi$. Hence the two factors $(1-B)$ and $(1+B)$ have to be present in $\alpha_3(B)$, so that $\alpha_3(B)= (1-B)(1+B)=1-B^2$. The model for z_{3t} is therefore given by

$$(1+B^2)z_{3t} = (1-B^2)a_{3t} \quad . \tag{1.16c}$$

Considering (1.16), it is seen that the canonical requirement has allowed us to identify the α-parameters of the component models. This is sufficient to identify fully the component models since, plugging the α-values in the system of autocovariance equations obtained before, the system becomes

$$\sigma_a^2 = 14\ \sigma_1^2 + 14\ \sigma_2^2 + 6\ \sigma_3^2 + 2\ \sigma_u^2$$

$$0 = 12\ \sigma_1^2 - 12\ \sigma_2^2$$

$$0 = 8\ \sigma_1^2 + 8\ \sigma_2^2 - 4\ \sigma_3^2$$

$$0 = 4\,\sigma_1^2 - 4\,\sigma_2^2$$

$$0 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_u^2$$

and a unique solution for the four unknown variances is obtained. This solution is given by

$$\sigma_1^2 = \sigma_2^2 = \sigma_a^2/64 \quad ,$$

$$\sigma_3^2 = \sigma_a^2/16 \quad , \tag{1.17}$$

$$\sigma_u^2 = 3\,\sigma_a^2/32 \quad .$$

Expressions (1.16) and (1.17) completely specify the models for the components; Figure 2 exhibits the z_{it}-component spectra. The first component, z_{1t}, obviously represents a trend component, and the white-noise u_t an irregular component. The other two components, z_{2t} and z_{3t}, contain the series variation for the frequencies $\omega=\pi$ and $\omega=\pi/2$, the twice-a-year and once-a-year seasonal frequencies in quarterly data.

The approach we have outlined provides elementary components, each one (except the irregular) unambiguously assigned to a peak in the series spectrum. In a second stage, the basic components can be aggregated as desired. Thus, in our example, the total seasonal component, s_t, would be

$$s_t = z_{2t} + z_{3t} \quad ,$$

and, replacing z_{2t} and z_{3t} with their expressions in (1.16), it is obtained that

$$(1+B)(1+B^2)s_t = (1+B^2)(1-B)a_{2t} + (1+B)(1-B^2)a_{3t} \tag{1.18}$$

The r.h.s. of (11) is an MA(3), say $\beta(B)\ c_t$, for which $\beta(1)=0$. Hence $\beta(B)$ contains the factor (1-B) and s_t will also be a canonical component. The total seasonal component will follow then the model,

$$(1+B+B^2+B^3)s_t = (1-B)(1-\beta_1 B-\beta_2 B^2)c_t \tag{1.19}$$

where c_t is white-noise. Equating the autocovariances of the r.h.s. of (1.18) and of (1.19) replacing σ_2^2 and σ_3^2 with their values given by (1.17), and solving for β_1, β_2 and σ_c^2, it is found that $\beta_1=-.819$, $\beta_2=-.344$, and $\sigma_c^2=.227\ \sigma_a^2$. The spectrum of s_t is given in Figure 3; the model obtained for s_t is, in this case, the same that would result from the seasonal adjustment method developed in Burman (1980) and Hillmer and Tiao (1982). Alternatively, the model for the seasonally adjusted series, z_t^a, can be obtained by summing the other two components z_{1t} and u_t, from which it is found that

$$\nabla\ z_t^a = (1+B)a_{1t} + (1-B)u_t\ .$$

Equating the autocovariances of the r.h.s. with those of an MA(1) process, the seasonally adjusted series is seen to follow the model

$$(1-B)z_t^a = (1-.42B)d_t, \tag{1.20}$$

with d_t white noise and $\sigma_d^2=.186\ \sigma_a^2$.

The previous discussion illustrates how, from the reduced form expression (model (1.9)), which can be obtained from the observations on z_t, it is possible to derive the structural model associated with that reduced form. If, in (1.9), ∇_4 is replaced by $(1-\Phi B^4)$, with $0<\Phi\leq 1$, and a_t is replaced by an invertible moving average $\theta_q(B)\ a_t$, with $q\leq 4$, the discussion remains basically unchanged. The trend, seasonal and irregular components would follow the models

$$(1-\Psi B)\ z_{1t} = (1+B)\ a_{1t}$$

$$(1+\Psi B+\Psi^2\ B^2+\Psi^3\ B^3)\ s_t = \beta_3(B)\ c_t$$

and u_t white-noise, with $\Psi = \Phi^{1/4}$, and the β-parameters and the variances σ_1^2, σ_c^2 and σ_u^2 being functions of the Φ- and θ-parameters. The series z_t will, in general, accept a perfectly sensible decomposition into trend, seasonal, and irregular components.

Let τ denote the number of observations in one year. The previous approach can be easily extended to other values of τ (such as, for example, τ=12) by introducing the appropriate new seasonal frequencies. This brings an interesting point: Models of the type

$$(1-\Phi\ B^{\tau})s_t = \theta(B)\ a_t \tag{1.21}$$

have been often employed to characterize the seasonal component. Examples can be found in Nerlove, Grether and Carvalho (1979), Pierce (1978), Pagan (1975), Engle (1978), Cleveland and Tiao (1976), Granger (1978), Harvey (1981), Ansley (1983), Gourieroux and Monfort (1983), and Pierce, Cleveland and Grupe (1984), among others. Although, in later work, some of these authors have changed the specification of the seasonal component, models that fit into (1.21) are still used, a recent example being Hausman and Watson (1985).

Since there is no generally accepted definition of what is a seasonal component, the choice of a model for such a component is, to some degree, arbitrary. Be that as it may, in the additive decomposition of z_t as in (1.11), the components z_{2t} and z_{3t} are clearly associated with seasonal variation, but in which way can $z_{1t} + u_t$ –or, equivalently, expression (1.20)– also be considered to represent seasonal variation?

Hausman and Watson (1985) state that, for their series, models that describe the seasonal by a set of dummy variables plus a stochastic component with an AR polynomial of the form $(1-\Phi B^{\tau})$ performed much better (in terms of their likelihood values) than models that used $(1+B+\ldots+B^{\tau-1})$ as the AR polynomial of the seasonal. Estimation criteria, however, cannot be used to decide, for a given series, whether the seasonal component should be characterized by, say, model (1.9) or (1.19). Having estimated model (1.9), it is always possible to decompose it as in (1.19) plus (1.20), and both representations will be observationally equivalent. As a consequence, the likelihood function cannot be of help in deciding which of the two representations should be used for the seasonal component. The decision depends on the implicit or explicit definition of this component, and it is difficult to accept a definition that includes as part of the seasonal a spectral peak for the (nonseasonal) frequency $\omega=0$, just as a spectral peak for the seasonal frequency $\omega=\pi$ would not be assigned to the trend.

In the final analysis, however, although a model such as (1.21) seems inadequate to characterize a seasonal component, the practical effect of this inadequacy is likely to be small since a series obeying (1.21) will be dominated by seasonal variation. As an example, the model

$$(1-.5B^{12})z_t = (1-.6B)a_t \quad ,$$

very close to the ones used by Hausman and Watson (1985) to characterize their stochastic seasonals, can be decomposed –following a reasoning similar to the previous one– into a purely seasonal component and a nonseasonal one consisting of the sum of a trend, given by

$$(1-.94\ B)p_t = (1+B)b_t \ ,$$

with $\sigma_b^2=.00095\ \sigma_a^2$, and a white-noise irregular with $\sigma_u^2=.0325\ \sigma_a^2$. To get an idea of the relative importance of the nonseasonal component, its standard deviation is found to be approximately equal to 19.2% of the standard deviation of z_t, a relatively small (although not negligible) percentage.

We have seen how to proceed in order to decompose an ARIMA model into unobserved components. Heuristically, components such as the trend or seasonal ones imply a mean that is not constant over time, displaying thus a nonstationary behavior, associated for each component with a particular frequency (and possibly its harmonics). This nonstationarity will be captured by unit autoregressive roots in the overall ARIMA model which shall determine then the autoregressive part of the component models. The moving average part of the component models will, on the one hand, impose the zeroes in the spectrum associated with the canonical property of not being contaminated by noise, and on the other hand, satisfy the "compatibility" condition (1.7), that ensures that the sum of the component models is compatible with the overall model for the observed series.

As mentioned before, the approach followed is easily extended to the case of series with different frequencies of observations. It can also be extended to estimate components associated with nonzero and nonseasonal frequencies. Consider, for example, a series that presents a nonstationary cycle of period T (larger than a year), revealed perhaps by the factorization of the autoregressive polynomial in the model for the observed series, or perhaps by a priori knowledge that such a cycle exists. One of the components of the series would follow then the model

$$(1-\Phi B+B^2)\, z_{it} = (1+B)(1-\beta B)\, a_{it}\ , \tag{1.22}$$

where $\Phi=2\cos\omega$ and $\omega=2\pi/T$. The model depends on two parameters: β and σ_i^2; Figure 4 displays the spectra of the component corresponding to a cycle with a period of 2 1/2 years in quaterly data, for $\beta = \pm.5$ and $\sigma_i^2=1$.

This example further illustrates the close relationship between the structural and reduced form approaches. (Components somewhat similar to (1.22) are used by Harvey (1985) within a structural approach.) The main difference between the two approaches possibly lies in the compatibility condition of the reduced form approach; in other words, on whether the information contained in the series should be used or ignored in the initical specification of the model. (For other differences and similarities, see Maravall, 1985.)

Having obtained sensible models for the components, it is then possible to perform estimation, diagnostic and inference in a rather natural way. We shall address these issues in the context of another example of more applied interest than the one considered in this section.

2. A "DEFAULT" MODEL AND AN EXAMPLE

Although the previous approach can handle more general decompositions, I shall concentrate on the usual decomposition of a series z_t into trend (p_t), seasonal (s_t) and irregular (u_t) components, as in

$$z_t = p_t + s_t + u_t \quad , \tag{2.1}$$

where the three components are independent. Often, the two components p_t and u_t are considered jointly, so that z_t is decomposed as in

$$z_t = z_t^a + s_t \ , \tag{2.2}$$

where $z_t^a = p_t + u_t = z_t - s_t$ is the seasonally adjusted series. Since u_t may be such that erratic short-term movements render the seasonally adjusted series a poor indicator of the underlying evolution of the series, trend estimation has often been recommended as an alternative or complement to seasonal adjustment. I consider, thus, separate estimation of p_t and u_t. By comparing the properties of the estimators of p_t and of z_t^a, some light will be shed on the relative virtues of using either of the two components.

In practice, at institutions such as the Bank of Spain, many hundreds of series are routinely decomposed as in (2.1) or (2.2), and it is impossible to perform the previous univariate analysis of each series. There is, thus, a need for a standard model that approximates reasonably well a large number of series and hence that can be applied routinely. Besides this practical reason, when dealing with a large collection of time series, there are also a priori theoretical reason for using some type of "common central model", perhaps letting just a few parameters differ across the series (see Sims, 1985).

An obviuos candidate among ARIMA models is the Airline model of Box and Jenkins (1970), given by

$$\nabla\nabla_{12} z_t = (1-\theta_1 B)(1-\theta_{12} B^{12})a_t , \qquad (2.3)$$

which has been found to approximate many series encountered in practice, characterized by the presence of trend and seasonal variation. The model (2.3) contains three parameters. Since $\theta_1 = 1$ implies a deterministic trend and $\theta_{12} = 1$ implies a deterministic seasonal component, θ_1 and θ_{12} are related to the stability of the trend and seasonal components, respectively. The third parameter, σ_a^2, provides a measure of the size of the one-period ahead forecast error.

When $-1<\theta_1<1$ and $0<\theta_{12}<1$, the model accepts a decomposition as in (2.1) (see Hillmer and Tiao, 1982). The discussion will be clearer if we focus on a particular example. Consider the monetary aggregate targeted in Spanish monetary policy: the series of liquid assets in the hands of the public (the sum of currency, deposits in banks and savings institutions, and other liquid assets). Estimation of (2.3) for the log of the monthly series, for the period 1973-1985 (T=156), yields $\theta_1=-.1915$ (SE=.080), $\theta_{12}=.6228$ (SE=.069), and $\sigma_a^2=.138\times10^{-4}$ (the standard error of the one-period-ahead forecast is aproximately equal to .37 percent of the level of the series.) The residual autocorrelation function (ACF) is relatively clean, and for example the Box-Pierce-

-Ljung statistics for the first 24 autocorrelations is equal to 20.6, well below the critical value $\chi^2_{22}(.05)=33.9$.

The spectrum of the estimated model for z_t is given in Figure 5, part a). It displays peaks for the frequencies $\omega=0$, associated with the trend, and $\omega=j\pi/6$, $j=1,\ldots, 6$, associated with the 1 to 6 times a year seasonal frequencies. Writing

$$\nabla\nabla_{12} = \nabla^2 S ,$$

where $S = 1+B+\ldots+B^{11}$, the peak for $\omega=0$ is induced by the factor ∇^2, while the peaks for $\omega=j\pi/6$, $j=1,\ldots,6$, are induced by the unit roots of S. (Notice that S can be expressed as

$$S = (1-\sqrt{3}B+B^2)(1-B+B^2)(1+B^2)(1+B+B^2)(1+\sqrt{3}B+B^2)(1+B) ,$$

where the six factors in the r.h.s. correspond to the one to six times a year seasonal frequencies.)

Therefore, the models for the trend, seasonal and irregular components will be of the type:

$$\nabla^2 p_t = \alpha(B)\ b_t$$
$$S\ s_t = \beta(B)\ c_t$$

and u_t white noise, where $\alpha(B)$ and $\beta(B)$ are polynomials in B of finite order. From (1.7), consistency with the overall model implies

$$\theta(B)\ a_t = S\alpha(B)\ b_t + \nabla^2\beta(B)c_t + \nabla\nabla_{12}\ u_t , \tag{2.4}$$

where $\theta(B) = (1-\theta_1 B)(1-\theta_{12} B^{12})$. Since the l.h.s. of (2.4) is a moving average of order 13, we can set $\alpha(B)$ to be of order 2 and $\beta(B)$ to be of order 11, so that the three terms in the r.h.s. are also of order 13. (It is, in fact, possible, to increase simultaneously the orders of

$\alpha(B)$ and $\beta(B)$ and still satisfy equation (2.4), but I shall not pursue this possibility any further.)

Equating the variance and autocovariances of the l.h.s. and r.h.s. of (2.4), a system of 14 equations is obtained. These equations express the relationship between the parameters of the overall model and the unknown parameters in the components models. Since the number of unknown parameters is 16 (α_1, α_2, $\beta_1,\ldots,\beta_{11}$, σ_b^2, σ_c^2, and σ_u^2), there there is an infinite number of structures of the type

$$\nabla^2 p_t = (1-\alpha_1 B - \alpha_2 B^2)b_t \tag{2.5a}$$

$$S\, s_t = (1-\beta_1 B - \ldots - \beta_{11} B^{11})c_t \tag{2.5b}$$

$$u_t \sim \text{white noise} \tag{2.5c}$$

that are compatible with the same reduced form given by model (2.3). One way to obtain, among these structures, the one with maximum irregular variance (i.e., the canonical decomposition) is the following:

Setting $\alpha_2=\beta_{11}=0$, the system of 14 covariance equations can be solved for the 14 unknowns. In the frequency domain, this solution is equivalent to the decomposition obtained in the first stage of the method developed by Burman (1980), which can be summarized as follows (see also Maravall and Pierce, 1987). Let $x=\cos\omega$ and denote by $U(x)/V(x)$ the spectrum of z_t in (2.3), where $U(x)$ and $V(x)$ correspond to the MA and AR parts of the model, respectively. Further, $V(x) = V_p(x)\ V_s(x)$, where $V_p(x)$ is associated with ∇^2 and $V_s(x)$ with S. Using a partial fractions decomposition, the spectrum of z_t can be expressed as

$$U(x)/V(x) = M_p(x)/V_p(x) + M_s(x)/V_s(x) + k\ , \tag{2.6}$$

where $k = \theta_1\ \theta_2$, and $M_p(x)$ and $M_s(x)$ are polynomials in x of order 1 and 10, respectively. Denote by $h_p^o(x)$ and $h_s^o(x)$ the first two terms in the r.h.s. of (2.6) and let $k_p = \min h_p^o(x)$ and $k_s = \min h_p^o(s)$ for $-1 \leq x \leq 1$. Then the spectra of the canonical components are given by

$$h_p(x) = h_p^o(x) - k_p$$

$$h_s(x) = h_s^o(x) - k_s$$

$$h_u(x) = k + k_p + k_s$$

Having obtained the spectra of the components, the autocovariance generating function (ACGF) can be computed and, by factorizing this function, the ARIMA component models can be derived.

For the seasonally adjusted series, the identity $z_t^a = p_t + u_t$ implies

$$\nabla^2 z_t^a = (1-\alpha_1 B-\alpha_2 B^2)\, b_t + \nabla^2 u_t\ , \tag{2.7}$$

where the r.h.s. is an MA(2). Therefore, z_t^a is an IMA(2,2) model, that shall be represented as

$$\nabla^2 z_t^a = (1-\lambda_1 B-\lambda_2 B^2)\, d_t\ , \tag{2.8}$$

where λ_1, λ_2 and σ_d^2 are functions of α_1, α_2, σ_b^2 and σ_u^2 which can be obtained by equating the variance and autocovariances of the r.h.s. of (2.7) and (2.8).

In order to analyse the decomposition of model (2.3) we can set $\sigma_a^2=1$. All variances will be expressed then in units of σ_a^2. For $\Theta_1 = -.1915$, $\Theta_{12} = .6228$, the models obtained for the components are:

$$\begin{aligned}\nabla^2 p_t &= (1+.039\ B-.961\ B^2)\ b_t \\ &= (1+B)(1-.961\ B)\ b_t\ ,\end{aligned} \tag{2.9a}$$

$$\nabla^2 z_t^a = (1-.779\ B-.175\ B^2)\ d_t$$
$$= (1+.182\ B)(1-.961\ B)\ d_t \tag{2.9b}$$

and

$$S\ s_t = (1+2.188\ B+2.581\ B^2+2.835\ B^3+2.689\ B^4+$$
$$+\ 2.28\ B^5+2.013\ B^6+1.388\ B^7+1.111\ B^8+$$
$$+\ .581\ B^9+.359\ B^{10}-.037\ B^{11})\ c_t\ . \tag{2.9c}$$

Futhermore the innovation variances are given by

$$\sigma_b^2 = .234\ ;\ \sigma_c^2 = .047\ ,$$
$$\sigma_u^2 = .108\ ;\ \sigma_d^2 = .670\ . \tag{2.10}$$

Thus, for example, the irregular component variance is approximately 10% of the one-step-ahead forecast error variance and hence the random character of the trend and seasonal components contributes heavily to the error in forecasting the overall series z_t.

The spectra of the three components, p_t, s_t and u_t are displayed in Figure 6. (The spectrum of z_t^a is that of p_t plus a constant.) The ACF of $\nabla^2 p_t$, Ss_t and u_t are given in Table 1. Looking at the factorization of $\alpha(B)$ in (2.9a), the root (1+B) induces the zero in the spectrum for $\omega=\pi$. The second root, (1-.961 B) is close to (1-B) and hence nearly cancels out with one of the ∇'s in the r.h.s. of (2.9a). Therefore, the model for the trend is similar to

$$\nabla\ p_t = (1+B)\ b_t + \delta\ ,$$

where δ is a constant, the only difference being that (2.9a) implies that δ changes very slowly over time. Similarly, the seasonally adjusted series follows a model close to

$$\nabla z_t^a = (1+.182\ B)\ d_t + \delta' \ ,$$

where, again, δ' is a constant.

The model for the seasonal component is a relatively complicated expression. The zero in the spectrum is attained at the frequency $\omega=.9175\pi$, between the 5 and the 6 times a year frequencies, the stationary component $S\ s_t$ displays a slowly decaying ACF.

Although the model for the trend depends on 3 parameters, the model for the seasonal component on 12, and the model for the irregular on 1, all those parameters are simply functions of θ_1 and θ_{12}. It is worth pointing out that different values of θ_1 and θ_{12} have very little effect on the α-parameters of the trend component model, and a moderate effect on the β-parameters of the model for the seasonal component. Figure 7 compares the ACFs of $\nabla^2 p_t$ and Ss_t when $\theta_1=-.1915$ and $\theta_{12}=.6228$ (the model we are analysing) with the (drastically different) case $\theta_1=.7$, $\theta_{12}=.2$. (In terms of the spectrum, the most noticeable difference is that, when $\theta_1<0$ the zero in the seasonal spectrum occurs between the seasonal frequencies $\omega=5\pi/6$ and $\omega=\pi$, while, when $\theta_1>0$, the zero occurs for $\omega=0$.) Different values of θ_1 and θ_{12}, however have a strong effect on the variance of the component model innovations. Thus, for $\theta_1=.7$, $\theta_{12}=.2$, it is obtained

$$\sigma_b^2 = .007 \ ; \ \sigma_c^2 = .147 \ ; \ \sigma_u^2 = .218$$

Comparing these values with those of (2.10), it is seen that more stable trends (i.e. larger values of θ_1) yield smaller values of σ_b^2, and more stable seasonal components (i.e., larger values of θ_{12}) yield smaller values of σ_c^2. Therefore, in terms of the structural parameters, different reduced form parameters (θ_1 and θ_{12}) translate mostly into differences in the variances of the component model innovations, leaving the rest of the structure relatively unchanged. Moreover, the more random a component is, the larger will be its innovation variance.

It should be pointed out that the previous discussion centers on the decomposition of the Airline model. If alternative specifications are used for the overall model, the effect on the components can be substantial; see Maravall (1986b).

3. ESTIMATION

3.1 Minimum Mean Squared Error Estimators

Having obtained the model for the components (the structure), it is then possible to proceed to estimation. One way to do it would be to write the structure in a state space format and use the Kalman filter. This method has been typically associated with the structural approach. I shall follow intead an alternative procedure, used mainly in conjunction with the reduced form approach. For the linear (Gaussian) processes we consider, the two methods are different ways of computing basically the same thing.

Back to the general framework of subsection 1.1, when the information consists of a complete realization of z_t, denoted by $[z_t]$, the minimum mean squared error (MMSE) estimator of the component z_{it} is given by

$$\hat{z}_{it} = k_i \frac{\Psi_i(B)\ \Psi_i(F)}{\Psi(B)\ \ \Psi(F)} z_t = \nu_i(B,F)\ z_t \tag{3.1}$$

where $k_i=\sigma_i^2/\sigma_a^2$ and $F=B^{-1}$. Under our assumptions, $\hat{z}_{it}$ is also equal to the conditional mean $E(z_{it}|[z_t])$. The derivation of (3.1) for the stationary case can be found in Whittle (1962), and the extension to nonstationary series in Cleveland and Tiao (1976) and Bell, (1984). (An extension of the Kalman Filter method to nonstationary series is contained in Burridge and Wallis, 1984.)

For ARIMA models, using (1.3), (1.5) and (1.8) in (3.1), the filter in the last expression becomes

$$\nu_i(B,F) = k_i \frac{\theta_i(B)\ \theta_i(F)\ \phi_i^*(B)\ \phi_i^*(F)}{\theta(B)\ \theta(F)} , \tag{3.2}$$

and $\hat{z}_{it}$ is given by

$$\hat{z}_{it} = \nu_i(B,F)\ z_t \tag{3.3}$$

$$= \ldots + \nu_2\ z_{t-2} + \nu_1\ z_{t-1} + \nu_o\ z_t + \nu_1\ z_{t+1} + \nu_2\ z_{t+2} + \ldots .$$

The filter $\nu_i(B,F)$ is centered and symmetric, and invertibility of the overall model guarantees its convergence. This convergence permits us to truncate the filter and apply in practice expression (3.3) to a finite time series.

Assume the truncated filter contains (2m+1) coefficients, so that

$$\hat{z}_{it} = \nu_o\ z_t + \Sigma_{j=1}^{m}\ \nu_j\ (z_{t-j} + z_{t+j}) \tag{3.4}$$

In order to obtain $\hat{z}_{it}$ by means of (3.3) observations from z_{t-m} to z_{t+m} are needed. As a consequence, at time T, when the available series is $z_1, \ldots, z_T$, estimation of z_{it} for t<m requires unavailable starting values of z, and estimation of z_{it} for t>T-m requires unknown future observations. Since

$$E_T\, z_{it} = E_T\, E(z_{it} \mid [z_t]) = E_T\, \hat{z}_{it} \;, \tag{3.5}$$

it follows that the estimator of z_{it} at time T can be computed by applying (3.3) to a series where each unavailable observation z_t has been replaced by the estimator $E_T\ z_t$. As an example, consider the concurrent estimator of z_{it} (the one obtained at time t, and naturally the case of most applied interest.) Assuming t>m, taking expectations at time t in (3.4), the concurrent estimator $\hat{z}^o_{it}$ can be computed as

$$\hat{z}^o_{it} = \nu_o\, z_t + \Sigma^m_{j=1}\, \nu_j(z_{t-j}+\hat{z}_t(j)), \tag{3.6}$$

where $\hat{z}_t(j) = E_t\ z_{t+j}$. That is, $\hat{z}^o_{it}$ is obtained by applying the filter $\nu_i(B,F)$ to the "extended series": $z_1, \ldots, z_t, \hat{z}_t(1), \ldots, \hat{z}_t(m)$. Consequently, in a particular application, both ends of the series of component estimators will be contaminated by errors in forecasting values of the series outside the sample period. I shall come back to this issue later on; for the present, we center our attention on the complete filter (i.e. in the final estimator (3.3)).

For the Airline model (2.3) and components models of the type (2.5), writing for notational simplicity:

$$\theta = (1-\theta_1\ B)(1-\theta_{12}\ B^{12})$$

$$\alpha = 1-\alpha_1\ B-\alpha_2\ B^2$$

$$\beta = 1-\beta_1\ B-\ldots-\beta_{11}\ B^{11} \;,$$

$$\lambda = 1-\lambda_1\ B-\lambda_2\ B^2 \;,$$

and letting a bar denote the same polynomial with B replaced by F, expression (3.2) becomes

$$\nu_p(B,F) = k_b\ \frac{\alpha\bar{\alpha}\ S\bar{S}}{\theta\ \bar{\theta}} \tag{3.7a}$$

for the trend component, and

$$\nu_s(B,F) = k_c \frac{\alpha\bar{\alpha}\ \nabla^2\bar{\nabla}^2}{\theta\ \bar{\theta}} \tag{3.7b}$$

for the seasonal component. In the example we are analysing (the monetary aggregate series), from (2.9) and (2.10), the two filters are easily obtained. They are shown in Figure 8.

The estimator of the irregular component is obtained as the residual, after the trend and sesonal component estimators have been removed. Hence

$$\hat{u}_t = z_t - \hat{p}_t - \hat{s}_t$$

$$= [1-\nu_p(B,F)-\nu_s(B,F)]z_t \ . \tag{3.8}$$

Taking $\nabla\nabla_{12}$ in both sides on (2.1) and considering (2.3) and (2.5), it is obtained that

$$\theta\ a_t = \alpha\ S\ b_t + \beta\ \nabla^2\ c_t + \nabla^2\ S\ u_t\ ,$$

from which

$$\theta\bar{\theta}\ \sigma_a^2 = \alpha\bar{\alpha}\ S\bar{S}\ \sigma_b^2 + \beta\bar{\beta}\ \nabla^2\bar{\nabla}^2\sigma_c^2 + \nabla^2\bar{\nabla}^2\ S\bar{S}\ \sigma_u^2 \tag{3.9}$$

Dividing by σ_a^2 in both sides and using (3.7), the identity (3.9) can be rewritten as

$$1-\nu_p(B,F)-\nu_s(B,F) = k_u \frac{\nabla\ \bar{\nabla}\ \nabla_{12}\bar{\nabla}_{12}}{\theta\ \bar{\theta}}\ ,$$

and (3.8) becomes

$$\hat{u}_t = k_u \frac{\nabla \bar{\nabla} \nabla_{12} \bar{\nabla}_{12}}{\theta \bar{\theta}} z_t . \tag{3.10}$$

Therefore, u_t estimated as the residual is the same as the one that would result from direct estimation using (3.1), equal to

$$\hat{u}_t = k_u [\Psi(B) \Psi(F)]^{-1} z_t .$$

More generally, it is irrelevant which two of the three components in the r.h.s. of (2.1) are estimated directly, leaving the third as the residual. Considering (2.2), a reasoning similar to the previous one shows that the estimator of the seasonally adjusted series computed as $\hat{z}^a_t = z_t - \hat{s}_t$ is the same as the one that would be obtained by direct estimation using (2.8).

Returning to the Spanish Money Supply, estimates of the components are displayed in Figure 9. (Notice that, once the spectra of the components are known, the autocovariance functions are easily derived, and nothing more is needed to find the filters ν_p and ν_s. In particular, estimation of the components does not require the derivation of their ARIMA expressions.)

3.2 <u>The Models for the Estimators</u>

Having derived expressions for the component models and for their MMSE estimators, by comparing the two it is seen that, as noticed by Grether and Nerlove (1970), the model for a component is different from the model for its estimator. It is of interest to look at the differences between the two.

An easy way to derive the theoretical model for the estimator is the following: Setting $z_t = \Psi(B) a_t$ in (3.1), the estimator $\hat{z}_{it}$ can be expressed as a function of the innovations $[a_t]$ in the observed series. After simplifying, it is obtained that

$$\hat{z}_{it} = \Psi_i(B)\ \eta_i(F)\ a_t\ , \tag{3.11}$$

where $\Psi_i(B)$ is the filter for the theoretical component in (1.2), given by $\Psi_i(B) = \theta_i(B)/\Phi_i(B)$, and

$$\eta_i(F) = k_i \frac{\theta_i(F)\ \Phi_i^*(F)}{\theta(F)}\ . \tag{3.12a}$$

Let

$$\hat{\Psi}_i(B,F) = \Psi_i(B)\ \eta_i(F) \tag{3.12b}$$

The filter $\hat{\Psi}_i$ is convergent in F and divergent in B. Its denominator implies that the estimator and the component require the same stationarity transformation and, due to the presence of $\theta_i(B)$ in the numerator, estimation preserves the canonical property of the component. The spectrum of the estimator, however, will display additional zeroes, corresponding to the unit roots of $\Phi^*(F)$ in the denominator of $\hat{\Psi}_i$. More generally, in terms of the ACF and of the spectral profile, the estimator will differ from the component because of the presence of $\eta_i(F)$ in (3.11). (It should be mentioned that if a series z_t follows the model (1.4), it also follows the model $z_t=\Psi_2\ (F)\ e_t$, where e_t is the backward innovation $e_t=z_t-E(z_t/z_{t+1},\ z_{t+2},\ \ldots)$, a white noise variable independent of all future z's. Corresponding to this formulation, $\hat{z}_{it}$ could be expressed alternatively in terms of the backward innovations as $\hat{z}_{it}=\Psi_i(F)\ \eta_i(B)\ e_t$.)

For the case of the Airline model, the expressions for the seasonal and trend estimators become

$$\nabla^2 \hat{p}_t = \alpha(B)\ \eta_p(F)\ a_t\ , \tag{3.13a}$$

$$S\ \hat{s}_t = \beta(B)\ \eta_s(F)\ a_t\ , \tag{3.13b}$$

where

$$\eta_p(F) = k_b\ \frac{\bar{\alpha}\ \bar{S}}{\bar{\theta}} \tag{3.14a}$$

$$\eta_s(F) = k_c\ \frac{\bar{\beta}\ \bar{\nabla}^2}{\bar{\theta}}, \tag{3.14b}$$

For the irregular component estimator

$$\hat{u}_t = \eta_u(F)\ a_t\ , \tag{3.15}$$

where

$$\eta_u(F) = k_u\ \frac{\bar{\nabla}\ \bar{\nabla}_{12}}{\bar{\theta}}, \tag{3.16}$$

and hence $\hat{u}_t=k_u\ \Psi(F)^{-1}\ a_t$. Notice that $\hat{u}_t$ is a linear function of future innovations a_{t+j}, $j\geq0$, so that although autocorrelated, it cannot be forecast. The concurrent estimator, however, will be white noise since $E_t\ \hat{u}_t=k_u\ a_t$ implies that it is a fraction of the concurrent innovation. The estimator $\hat{u}_t$ is seen to be stationary, with finite variance. This variance is always smaller than that of the theoretical u_t; in fact, the smaller this last variance is, the larger (in relative terms) will be the underestimation of the variance (see Maravall, 1987).

From expressions (3.13) to (3.16), ACFs and spectra of the theoretical estimators can be easily computed. For the example of the monetary aggregate series they are displayed in Figures 10 and 11, where they are compared to those of the theoretical components. Looking at the ACFs, it is seen that, for the seasonally adjusted series, MMSE estimation leaves practically unchanged the low order autocorrelations, while it induces some negative autocorrelation at lag 12. In the case of the trend, estimation changes ρ_1 from .18 to .00 and also induces some negative autocorrelation at lag 12. For the seasonal component, the slow decay of the component ACF is replaced by a cycle of period 12 for the ACF of the estimator. In the case of the irregular component, the two ACFs differ markedly, with the estimator displaying negative autocorrelation at both low-order and seasonal lags. Finally, the negative autocorrelations at lag 12 induced in the seasonally adjusted, trend, and irregular components are seen to be all equal to -.19.

The spectra of the estimators possess zeroes that are implied by the unit roots in the denominators of the $\hat{\Psi}_i$ filters. Let $\hat{g}_p(\omega)$, $\hat{g}_s(\omega)$ and $\hat{g}_u(\omega)$ denote the spectra of the trend, seasonal and irregular component estimators, respectively. The zeroes in $\hat{g}_p(\omega)$ and $\hat{g}_u(\omega)$ for the seasonal frequencies reflect the fact that, for these frequencies, the ratio of the variance of the trend and of the irregular component to that of the seasonal component is zero. Therefore, in the spectrum of z_t, these frequencies will be ignored when estimating the trend or the irregular component. Similarly, the zero in $\hat{g}_s(\omega)$ and $\hat{g}_u(\omega)$ for $\omega=0$ is explained by the fact that, for $\omega=0$, the ratio of the variance of the seasonal or the irregular to that of the trend is zero.

In relative terms, the difference between the two spectra is particularly noticeable for the case of the irregular component estimator, which is far for being white noise. Its upward shape reflects the predominance of the trend component variance as the frequency becomes lower.

In all cases, the spectrum of the estimator lies below the spectrum of the component. Accordingly, the variance of the (stationary transformation) of the estimator is smaller than that of the component, as seen in Table 2. Since the sum of the three components is equal to the sum of the three estimators, the difference in the sum of the variances reflects covariances among the estimators. While the theoretical components are uncorrelated, the estimators $\nabla^2\hat{p}_t$, $S\hat{s}_t$ and $\hat{u}_t$, in view of (3.13) and (3.15), will be correlated in general. Writing

$$\alpha(B)\ \eta_p(F) = \Sigma_{j=-2}\ \hat{\alpha}_j\ F^j$$

$$\beta(B)\ \eta_s(F) = \Sigma_{j=-11}\ \hat{\beta}_j\ F^j$$

and

$$\eta_u(F) = \Sigma_{j=0}\ \hat{\eta}_j\ F^j$$

the covariances are given by

$$E(\nabla^2\hat{p}_t\ S\hat{s}_t) = \Sigma_{j=-11}\ \hat{\alpha}_j\ \hat{\beta}_j\ ,$$

$$E(\nabla^2\hat{p}_t\ \hat{u}_t) = \Sigma_{j=-2}\ \hat{\alpha}_j\ \hat{\eta}_j\ ,$$

and

$$E(S\hat{s}_t\ \hat{u}_t) = \Sigma_{j=-11}\ \hat{\beta}_j\ \hat{\eta}_j$$

where $\hat{\alpha}_{-11}=\ldots=\hat{\alpha}_{-3}=\hat{\eta}_{-11}=\ldots=\hat{\eta}_{-1}=0$. Proceeding in this way it is found that, for the example we consider,

$$\begin{aligned} \text{Corr}\ (\nabla^2\hat{p}_t,\ S\hat{s}_t) &= .10 \\ \text{Corr}\ (\nabla^2\hat{p}_t,\ \hat{u}_t) &= .06 \\ \text{Corr}\ (S\hat{s}_t,\ \hat{u}_t) &= .05\ . \end{aligned}$$

Thus, although nonzero, the correlations between the estimators are nevertheless small.

4. DIAGNOSIS AND INFERENCE

4.1 Diagnosis

An important virtue of a model-based approach to unobserved component estimation is that it provides the grounds for diagnostic checks by comparing theoretical models with the obtained estimates. As we have seen, the theoretical model considered in the comparison should be that of the estimator, which can be quite different from that of the theoretical component.

Figure 12 exhibits the ACF of the stationary transformations of the theoretical estimators, derived from (3.13) and (3.15), and compares them with the empirical ACF of the component estimates for the monetary aggregate series. For the seasonally adjusted series and the trend and irregular components, the empirical and theoretical ACFS are in close agreement. In the case of the seasonal component, the shapes are also similar, although the empirical ACF dies off faster that the theoretical one.

In order for the comparison of the two ACF to be meaningful, we need to have an idea of how close we can expect to get to the theoretical autocorrelations in a particular realization. To answer that question, three hundred independent series were generated with the Airline model with $\theta_1=-.1915$ and $\theta_{12}=.6228$. Each series consisted of 156 observations. The trend, seasonal and irregular components were estimated and the variance and ACF were computed for their stationary transformations. As was mentioned before, the series of estimates obtained are contaminated at both ends by the replacement of starting and future observations by expectations. We found, however, that the results changed very little when years were removed from both ends of the series, and the results reported are for the complete series of estimates.

The biases were found to be small, practically nonexistent for ρ_1 and for the variance, and slowly increasing for ρ_k as k gets larger. Table 3 reports the results for the estimators of ρ_1, ρ_{12} and the standard deviation (σ) of the stationary transformation of the component estimators. Table 3 displays also their theoretical values and the empirical estimates obtained for the monetary aggregate series.

The estimator of ρ_1 appears to be reasonably unbiased, with a standard deviation in the order of .06, except for the seasonal component, where the estimator is particularly accurate. As for the estimator of ρ_{12}, it displays a small bias and a standard deviation of .08, except for the seasonal component, in which case the estimator is considerably more innacurate. Finally, the estimator of the standard deviation of the stationary component estimator is fairly well behaved in all cases, with hardly any bias and a small standard deviation (particularly small for $\hat{u}_t$).

The comparison between the last two rows in a), b) and c) of Table 3 provides an overall check of the validity of the decomposition obtained. Considering the simulation results, the estimates obtained for the seasonally adjusted series, trend and irregular components are comfortably in agreement with the theoretical estimators. For the seasonal component, however, both ρ_{12} and σ are borderline acceptable.

A similar simulation was carried out for a series half the length of the one considered in our example. The estimators had small bias (although the bias of ρ_{12} increased slightly) and were reasonably precise. As an example, for the irregular component, the mean and standard deviations of ρ_1, ρ_{12} and σ were, respectively $-.59$ (standard deviation = .09), $-.24$ (standard deviation = .10), and .18 (standard deviation = .02). Comparison of the theoretical and empirical second moments of the stationary component estimators seems to provide a convenient, easy to compute, check on the results. In the example we are considering, the check leads to (nonenthusiastic) acceptance of the results.

4.2. Inferences

An important issue of applied concern (see for example, Bach et al., 1976, and Moore et al, 1981) is the error incurred in estimating the components. In the example we are considering of the Spanish monetary aggregate, since monthly targets are set for the seasonally adjusted series, in order to judge whether targets are being met or not it is important to know how accurately the seasonally adjusted series can be measured. Furthermore, the measurement error in the adjusted series should imply a range of tolerance for future targets. The model-based approach offers a convenient framework to address the issue (see, for example, Pierce, 1979 and 1980, Hillmer, 1985, and Burridge and Wallis, 1985).

There are several types of errors involved in estimation of the components. Consider the estimator $\hat{z}_{it}$ given by (3.1). This is the final estimator of z_{it}, obtainable when a complete realization of z_t is available. The error

$$\delta_{it} = z_{it} - \hat{z}_{it} \tag{4.1}$$

will be called the "final estimation error". The second type of error is related to the distorssion induced at both ends of the component estimator series by the fact that starting and future values of the observed series are unknown, as mentioned in Subsection 3.1. Direct inspection of the filters ν_p and ν_s in Figure 8 shows that the weight assigned to the observation z_T in the estimation of a component, s_t or p_t, is negligible when T and t are separated by more than five years (this is also evident from the results in Hillmer, 1985.) Considering the series length, it follows that the unknown starting values will only affect the early, distant years. We focus on the error induced by the lack of future observations, which shall be termed "revision error". As shown in Pierce (1980), the final estimation and the revision errors are independent of each other, therefore I shall analyze the two separately.

a) Final Estimation Error

For notational simplicity, let u_t be included as an additional z_{it}-component (for which $\Phi_i(B)=\theta_i(B)=1$), and consider estimation of the first component. Equation (4.1) can be rewriten as

$$\delta_{1t} = [1-\nu_1(B,F)]z_{1t} - \nu_1(B,F)\, \Sigma_{j=2}\, z_{jt} ,$$

where $\nu_1(B,F)$ is given by (3.2), and using the ARIMA expressions for the components, it is obtained that

$$\delta_{1t} = (1-\nu_1)\, \frac{\theta_1}{\Phi_1}\, a_{1t} - \nu_1 \sum_{j=2} \frac{\theta_j}{\Phi_j}\, a_{jt} \tag{4.2}$$

where the greek letters denote polynomials in B and in F. Therefore, δ_{1t} can be expressed as

$$\delta_{1t} = \Sigma_j\, \zeta_j\, a_{jt} , \tag{4.3}$$

where ζ_j denotes the polynomials in the r.h.s. of (4.2). Notice that δ_{1t} is the difference between two nonstationary series. This is reflected in the presence of Φ_1 and Φ_j in the denominators of the r.h.s. of (4.2), which would seem to indicate nonconvergence of the polynomial ζ_i and hence that δ_{1t} is nonstationary. This is not the case, however, as we proceed to show.

Using (3.2) for i=1 and letting, as before, a bar denote the corresponding polynomial in F,

$$1-\nu_1 = \frac{\theta\bar{\theta} - k_1\theta_1\bar{\theta}_1\phi_1^{*}\bar{\phi}_1^{*}}{\theta\bar{\theta}}$$

But, from $z_t = \Sigma\, z_{jt}$, multiplying through by Φ, the following identity for the ACGF is obtained

$$\theta\bar{\theta} - k_1\theta_1\bar{\theta}_1\phi_1^{*}\bar{\phi}_1^{*} = \Sigma_{j=2}\, \theta_j\bar{\theta}_j\phi_j^{*}\bar{\phi}_j^{*}\, k_i .$$

Considering (1.8), ϕ_j^{*} for j=1 contains ϕ_1. Hence factorizing ϕ_j^{*} as $\phi_j^{*} = \phi_1\, \phi_{1j}^{*}$, so that

$$\phi_{1j}^{*} = \pi_{k\neq j,1}\, \phi_j ,$$

the polynomial ζ_1 can be finally expressed as

$$\zeta_1 = \frac{\theta_1}{\theta\bar{\theta}} \left[\underset{j=2}{\Sigma}\, \theta_j\bar{\theta}_j\phi_{1j}^{*}\bar{\phi}_j^{*}\, k_j \right]$$

and hence is a convergent filter in B and in F. For j=1, since then ϕ_1^{*} contain ϕ_j, the polynomial ζ_j is given by

$$\zeta_j = k_1\, \frac{\theta_1\bar{\theta}_1\phi_{1j}^{*}\bar{\phi}_1^{*}\theta_j}{\theta\bar{\theta}} ,$$

a filter that converges also in B and in F.

The properties of δ_{1t} can be derived alternatively as follows. Rewrite (1.1) as

$$z_t = z_{1t} + Z_{1t} ,$$

where $Z_{1t} = \Sigma_{j=2} z_{jt}$. Since the components follow ARIMA models, Z_{1t} will also be an ARIMA and its expression will be of the type

$$\phi_1^* Z_{1t} = \theta_1^* g_t ,$$

where g_t is white noise with variance σ_g^2, ϕ_1^* is as in (1.8) and θ_1^* is a moving average that can be obtained by factorizing the ACF of $\Sigma_{j=2} z_{jt}$. Then it can be seen that (4.3) simplifies into

$$\delta_{1t} = \frac{\theta_1 \theta_1^*}{\theta} \varepsilon_t , \tag{4.4}$$

where ε_t is white noise, with $\sigma_\varepsilon^2 = \sigma_1^2 \sigma_g^2 / \sigma_a^2$ (see Pierce, 1979). Notice that the final estimation error for all components is an ARMA process, with the autoregressive polynomial always the same and equal to the moving average polynomial of the model for the observed series.

For the Airline model and the three-component decomposition (2.1), since it has to be that $\Sigma_i \delta_{it} = 0$, we need to consider only two out of the three components' final estimation errors. The trend final estimation error is given by the expression

$$\delta_{pt} = \zeta_1 b_t + \zeta_2 c_t + \zeta_3 u_t , \tag{4.5}$$

where, using the notation of Section 3.1,

$$\zeta_1 = \frac{\alpha \bar{\nabla}^2 \beta \bar{\beta}}{\theta \bar{\theta}} k_c + \frac{\alpha \bar{\nabla}^2 S \bar{S}}{\theta \bar{\theta}} k_u ,$$

$$\zeta_2 = - \frac{\alpha \bar{\alpha} \beta \bar{S}}{\theta \bar{\theta}} k_b$$

and

$$\zeta_3 = - \frac{\alpha\bar{\alpha}\ S\bar{S}}{\theta\bar{\theta}} k_b \ .$$

These filters can be computed for the component models (2.9), and (4.5) can be used to find the variance of the final estimation error. Equation (4.5) permits us to split this variance into three parts associated with the trend, seasonal and irregular component innovations.

A similar derivation for the seasonal component, or equivalently, the seasonally adjusted series, yields an expression of the type

$$\delta_{at} = \xi_1\ c_t + \xi_2\ d_t \quad , \tag{4.6}$$

where $\delta_{at} = z^a_{it} - \hat{z}^a_{it}$, and with

$$\xi_1 = \frac{\lambda\bar{\lambda}\ \beta\ \bar{S}}{\theta\bar{\theta}} k_d$$

$$\xi_2 = - \frac{\lambda\bar{\nabla}^2\ \beta\bar{\beta}}{\theta\bar{\theta}} k_c \ .$$

For the models (2.9) and (2.10), expression (4.6) can be used to compute the variance of δ_{at} and to decompose it into parts associated with the seasonal and nonseasonal component innovations.

The result are shown in Table 4. Most of the error in the final estimator of the trend is caused by the randomness of the trend component innovation. In the case of the seasonally adjusted series, the seasonal and nonseasonal component innovations have contributions of a similar magnitude.

In terms of (4.4), the model for the final estimation error in the seasonally adjusted series can be expressed as

$$\theta\, \delta_{at} = \beta\, \lambda\, \varepsilon_t \ .$$

Therefore δ_{at} follows a stationary ARMA (13, 13) model with autoregressive polynomial $(1-\theta_1 B)(1-\theta_{12} B^{12})$. For our example, the ACF is displayed in the first column of Table 5, and it is seen to be strongly seasonal.

Table 4 shows that the final estimation error of the trend and of the seasonally adjusted series have similar variances. In both cases, the standard deviation of the error is close to one half of the standard deviation of the one-period-ahead forecast error of the overall series. In our example, the error in the final estimator of the seasonally adjusted series is of considerable size; no improvement however in precision can be expected from using, alternatively, the trend.

b) Revision Error

As we saw in subsection 3.1, in order to obtain $\hat{z}_{it}$ by means of (3.3), a complete realization $[z_t]$ is needed. As a consequence, at time T, when the last observation available is z_T, estimation of z_{it} for t close enough to T (in practice, using the truncated filter (3.4), for t>T-m) requires unknown future observations. As already mentioned, a preliminary estimator can be obtained by applying (3.3) to the extended series: $z_1, \ldots, z_T, \hat{z}_T(1), \hat{z}_T(2) \ldots$, where $\hat{z}_T(j)$ denotes the forecast of z_{T+j} with origin T. Accordingly, the preliminary estimator will be subject to revisions since, as new observations become available, forecasts will be updated and eventually replaced with observations. The difference between the preliminary and final estimators represents a measurement error in the former and will be called the revision error.

Consider first the estimation of z_{it} at time t. In view of (3.6) the revision in the concurrent estimator, $\hat{z}^o_{it}$, is

$$r^{o}_{it} = \hat{z}_{it} - \hat{z}^{o}_{it} = \Sigma^{\infty}_{j=1}\ \nu_j (z_{t+j} - \hat{z}_t(j)) = \Sigma^{\infty}_{j=1}\ \nu_j\ e_t(j) \quad , \qquad (4.7)$$

where $e_t(j)$ denotes the j-th period-ahead forecast error of z_t. Since

$$e_t(j) = a_{t+j} + \Sigma^{j-1}_{k=1}\ \psi_k\ a_{t+j-k} \quad ,$$

expression (4.7) can be rewriten as a moving average of future innovations a_{t+1}, a_{t+2},... However, a more direct way of obtaining this moving average representation is through the model derived for the estimators in Subsection 3.2, equal to

$$\hat{z}_{it} = \hat{\Psi}_i(B,F)\ a_t = \Sigma^{\infty}_{j=-\infty}\ \hat{\Psi}_{ij}\ a_{t+j} \quad , \qquad (4.8)$$

where $\hat{\Psi}_i(B,F)$ is given by (3.12b). Since $E_t\ a_{t-j} = a_t$ for $j \geq 0$, and $E_t\ a_{t+j} = 0$ for $j>0$, it follows that

$$\hat{z}^{o}_{it} = E_t\ \hat{z}_{it} = \Sigma^{o}_{j=-\infty}\ \hat{\Psi}_{ij}\ a_{t+j} \quad , \qquad (4.9)$$

Substracting (4.9) from (4.8),

$$r^{o}_{it} = \Sigma^{\infty}_{j=1}\ \hat{\Psi}_{ij}\ a_{t+j} = \hat{\Psi}_{i1}\ a_{t+1} + \hat{\Psi}_{i2}\ a_{t+2} + \ldots \qquad (4.10)$$

$$= \hat{\Psi}_i(F)\ a_{t+1}.$$

From expression (4.10) it is possible to derive properties of the revisions. As an example, the revision in $\hat{u}^{o}_{t}$ in the decomposition of (1.4) into (1.1) can be seen to follow the model (see Maravall, 1986a).

$$\theta(F)\ r^{o}_{ut} = k_u[\Phi(F) - \theta(F)]a_t \ .$$

In a similar way, (4.8) can be used to derive the revision in any preliminary estimator. If $\hat{z}^{n}_{it}$ denotes the estimator of z_{it} obtained at time t+n ($n \gtrless 0$), then

$$\hat{z}^{n}_{it} = E_{t+n}\, \hat{z}_{it} = \Sigma^{n}_{j=-\infty}\, \hat{\Psi}_{ij}\, a_{t+j} \; .$$

and the revision in the preliminary estimator becomes

$$r^{n}_{it} = \hat{z}_{it} - \hat{z}^{n}_{it} = \Sigma^{\infty}_{j=n+1}\, \hat{\Psi}_{ij}\, a_{t+j} \; . \qquad (4.11)$$

Given that the filter $\hat{\Psi}_i(B,F)$ is convergent in F, the revision is a stationary process. Notice that (4.11) implies that the change in the revision when the estimation period moves from T to T+n is a moving average of order (n-1) (see Pierce, 1980), and that updating an estimator when a new observation becomes available is equivalent to adding the last innovation multiplied by the corresponding Ψ_i-weight.

For the Airline model, from (3.13),

$$\hat{\Psi}_p(B,F) = \frac{\alpha(B)\ \eta_p(F)}{\nabla^2} \qquad (4.12a)$$

$$\hat{\Psi}_s(B,F) = \frac{\beta(B)\ \eta_s(F)}{S} \; , \qquad (4.12b)$$

where $\eta_p(F)$ and $\eta_s(F)$ are given in (3.14). For the particular example we are considering (the models in (2.9)-(2.10)), the filters $\hat{\Psi}_p(B,F)$ and $\hat{\Psi}_s(B,F)$ can be obtained and the variance and autocorrelations of the revisions computed. Table 6 displays the variance of the revision in the concurrent estimator and of the revision after one, two, three, four and five years of additional data have become available. It is seen that, after five years, the revision in the trend and in the seasonally adjusted series are negligible, so that the filter (3.2) can be truncated safely. (In fact, more than 95% of the variance of the revision in the concurrent estimator is explained by the first three years.)

The revision error in the concurrent estimator of the trend is seen to be similar to, and slightly larger than, the revision in the concurrent estimator of the seasonal. Comparing Table 6 with Table 4, the variance of the final estimation error is also similar to that of the revision error, although the latter is slightly larger. Approximately, the revision error of the concurrent seasonal and trend estimators and the final estimation error of both components are all in the same order of magnitude: in the four cases, the standard deviation of the error is close to 50% of the standard deviation of the innovations in the observed series.

Until very recently, seasonal adjustment of the Spanish monetary aggregate series was done once a year instead of concurrently. This implied the use of the concurrent seasonal estimator for January and the 11-month ahead projected seasonal component for December. The variances of the revision error in the projected components can be computed through (4.11), with $\hat{\Psi}_s(B,F)$ replaced by (4.12b), and they are reported in Table 7. There is unquestionably some gain from using concurrent adjustment.

One implication of the previous results is the following. In connection with the conduct of short term monetary policy, Maravall and Pierce (1986) recently concluded "... why so much emphasis on seasonal adjustment? Perhaps attention should shift to estimation of a smoother signal less affected by revisions (possibly some type of trend)." For the case of the Spanish monetary aggregate the trend certainly provides a smoother signal, but it is not subject to smaller revisions, nor is it estimated with more precision. In fact, it can be seen that if projected (instead of concurrent) components are used, the revision error variance of the trend component estimator is much larger than that of the seasonal component estimator.

c) A Final Remark

Since targets are usually set for the monthly rate of growth of the seasonally adjusted monetary aggregate, it is of interest to see the effect of measurement error on that rate. Since the logs of the series are being used, the "true" rate, expressed in percentage points of annualized growth, can be approximated by $y_t = 1200\, \nabla z_t^a$, and the measured rate by $\hat{y}_t = 1200\, \nabla \hat{z}_t^a$. Writing z_t^a as $\hat{z}_t^a + \varepsilon_t$, where ε_t is the measurement error on z_t^a, the error in $\hat{y}_t$ is found equal to $v_t = y_t - \hat{y}_t = 1200(\varepsilon_t - \varepsilon_{t-1})$, from which

$$\mathrm{Var}(v_t) = (1-\rho_1) \times \mathrm{Var}(\varepsilon_t) \times 288 \times 10^4 ,$$

where ρ_1 is the lag-one autocorrelation of ε_t. When ε_t is the final estimation error, its ACF appears in Table 5, first column. For the revision error, the ACF can be derived from (4.10) and it is shown in the second column of Table 5. (The ACFs of the two errors are remarkably similar.) Using the values of ρ_1 and the variances of δ_t and r_t^o, it is found that, for the concurrent estimator $(\varepsilon_t = \delta_t + r_t^o)$, $\mathrm{Var}(v_t) = 7.23 = (2.69)^2$. For the final estimator, $\mathrm{Var}(v_t) = 3.23 = (1.77)^2$. Thus an approximate 95% confidence interval for y_t based on the concurrent estimator is given by $\hat{y}_t^o \pm 2 \times 2.69 = \hat{y}_t^o \pm 5.38$. When the final estimator becomes available the interval narrows to $\hat{y}_t \pm 2 \times 1.77 = \hat{y}_t \pm 3.54$. Therefore, based on the measurement error, the implicit tolerance range for the next month target should have a width of approximately 10 percentage points, larger than the tolerance ranges typically used in practice.

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Table 1

ACF of the theoretical components

Lag	$\nabla^2 z_t^a$	$\nabla^2 p_t$	$S\ s_t$	u_t
1	-.39	.001	.95	–
2	-.11	-.499	.84	–
3	–	–	.70	–
4	–	–	.54	–
5	–	–	.39	–
6	–	–	.26	–
7	–	–	.15	–
8	–	–	.08	–
9	–	–	.03	–
10	–	–	.01	–
11	–	–	.00	–

Table 2

Standard Deviation of Components

	$\nabla^2 z^a_t$	$\nabla^2 p_t$	$S\ s_t$	u_t
Theoretical Component	1.05	.67	1.38	.33
Theoretical Estimator	.94	.48	.34	.19

Table 3

Component Moments: Simulation

a) Lag-1 autocorrelation (ρ_1)

	$\nabla^2 \hat{z}^a_t$	$\nabla^2 \hat{p}_t$	$S \hat{s}_t$	$\hat{u}_t$
Simulation (Standard Error)	-.39 (.07)	.18 (.06)	.84 (.02)	-.59 (.06)
Theoretical Estimator	-.40	.18	.83	-.60
Estimate	-.44	.21	.81	-.64

b) Lag-12 autocorrelation (ρ_{12})

	$\nabla^2 \hat{z}^a_t$	$\nabla^2 \hat{p}_t$	$S \hat{s}_t$	$\hat{u}_t$
Simulation (Standard Error)	-.22 (.08)	-.22 (.08)	.52 (.16)	-.22 (.08)
Theoretical Estimator	-.19	-.19	.62	-.19
Estimate	-.19	-.27	.31	-.18

c) Standard Deviation

	$\nabla^2 \hat{z}^a_t$	$\nabla^2 \hat{p}_t$	$S \hat{s}_t$	$\hat{u}_t$
Simulation (Standard Error)	.96 (.04)	.49 (.03)	.33 (.06)	.19 (.01)
Theoretical Estimator	.94	.48	.34	.19
Estimate	.90	.44	.22	.18

Table 4

Variance of the Final Estimation Error

	Stemming from				Total
	b_t	c_t	u_t	d_t	
Trend	.103	.077	.038	–	.218
Seasonally Adjusted Series	–	.095	–	.105	.200

Table 5

ACF of Estimation Error:
Seasonally Adjusted Series

Lag	Final Estimation Error	Revision Error
1	.60	.52
2	.28	.34
3	-.03	.03
4	-.25	-.17
5	-.38	-.31
6	-.43	-.38
7	-.41	-.39
8	-.32	-.34
9	-.16	-.21
10	.07	.04
11	.32	.20
12	.63	.63

Table 6

Variance of Revision Error

	r_t^o	r_t^{12}	r_t^{24}	r_t^{36}	r_t^{48}	r_t^{60}
Revision in Trend	.240	.065	.025	.009	.004	.001
Revision in Seasonally Adjusted Series	.217	.085	.033	.012	.005	.002

Table 7

Variance of Revision Error:
Seasonally Adjusted Series

Revision in Forecasted Component	Variance
1-month-ahead	.249
2 " "	.270
3 " "	.290
4 " "	.300
5 " "	.304
6 " "	.304
7 " "	.305
8 " "	.307
9 " "	.314
10 " "	.334
11 " "	.349
12 " "	.400

Spectrum of $\nabla_4 z_t = a_t$

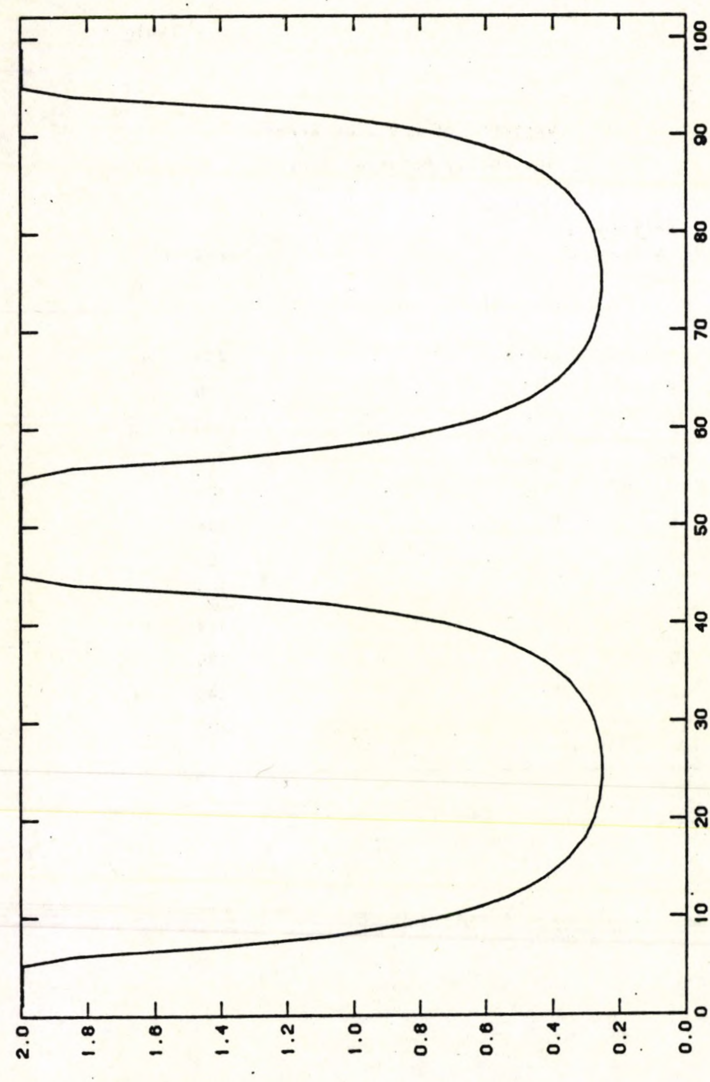


Figure 2

Components of $\nabla_4 z_t = a_t$

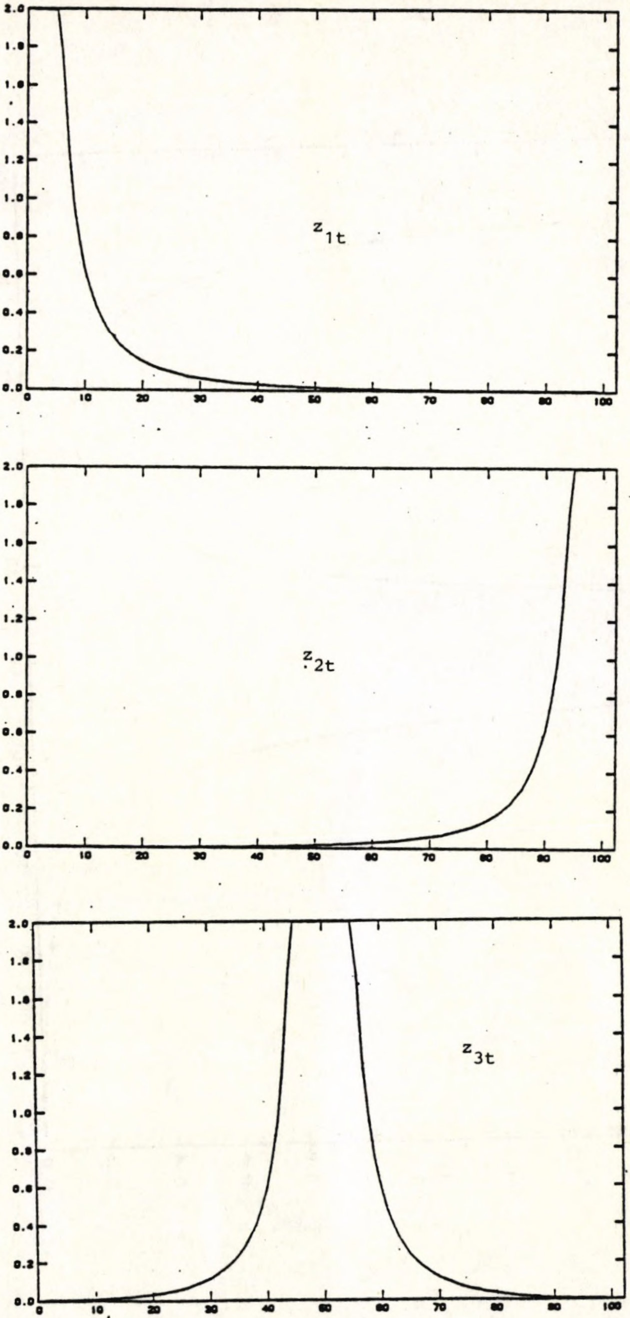
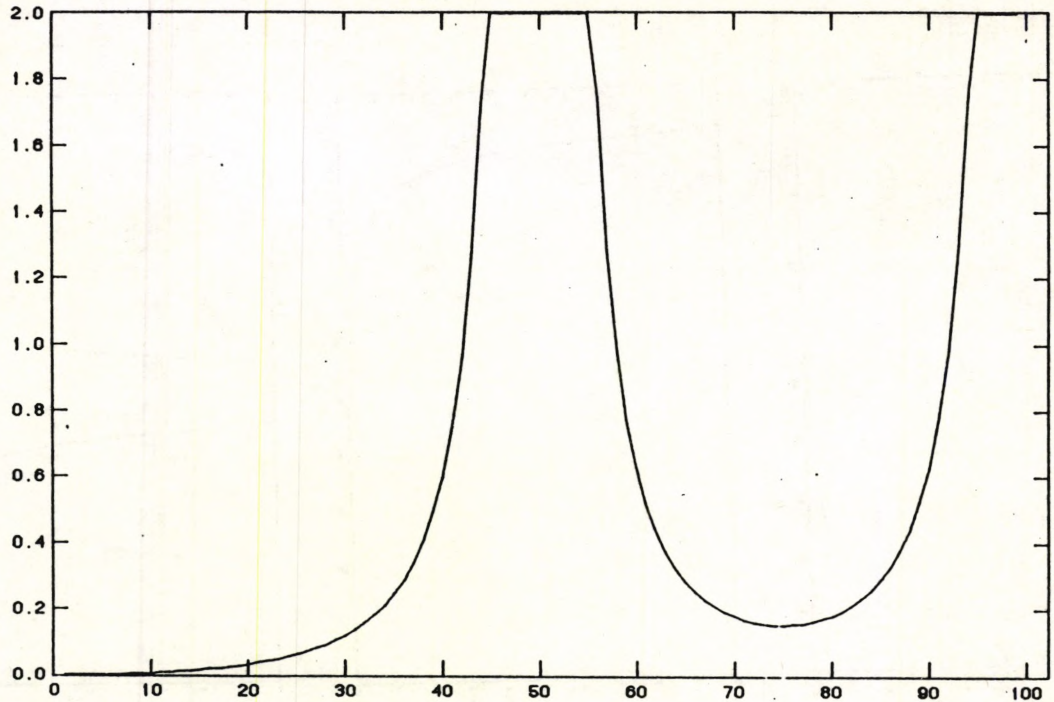


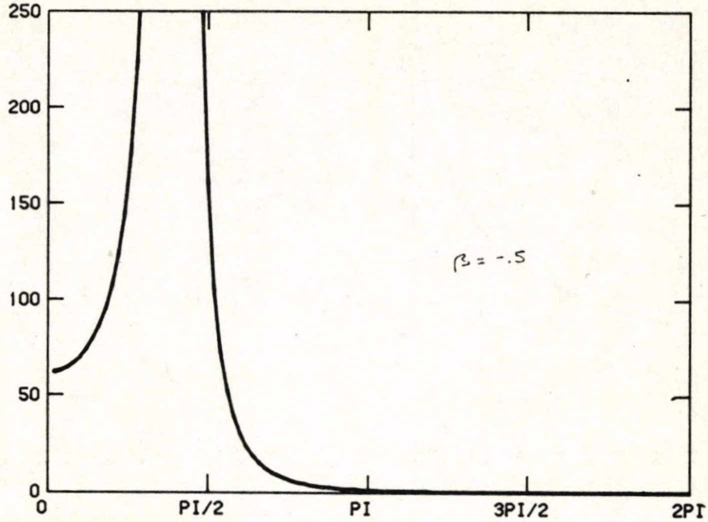
Figure 3

Total Seasonal Component



<u>Figure 4</u>

Spectrum of Cyclical Component



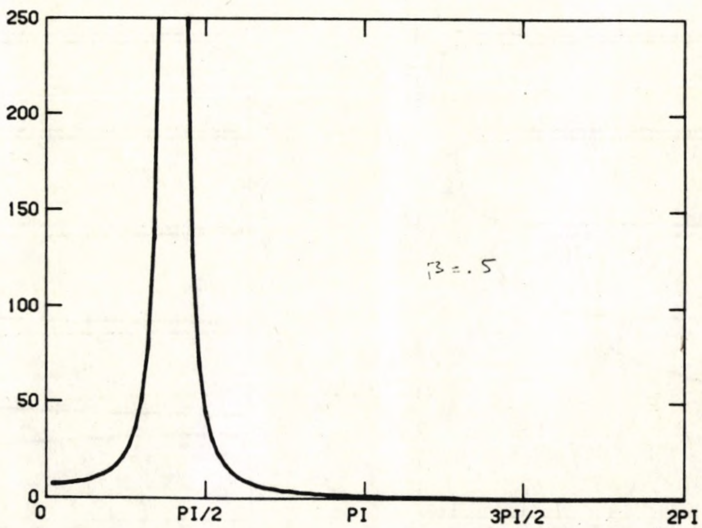


Figure 5

Spectra of Series and of Components

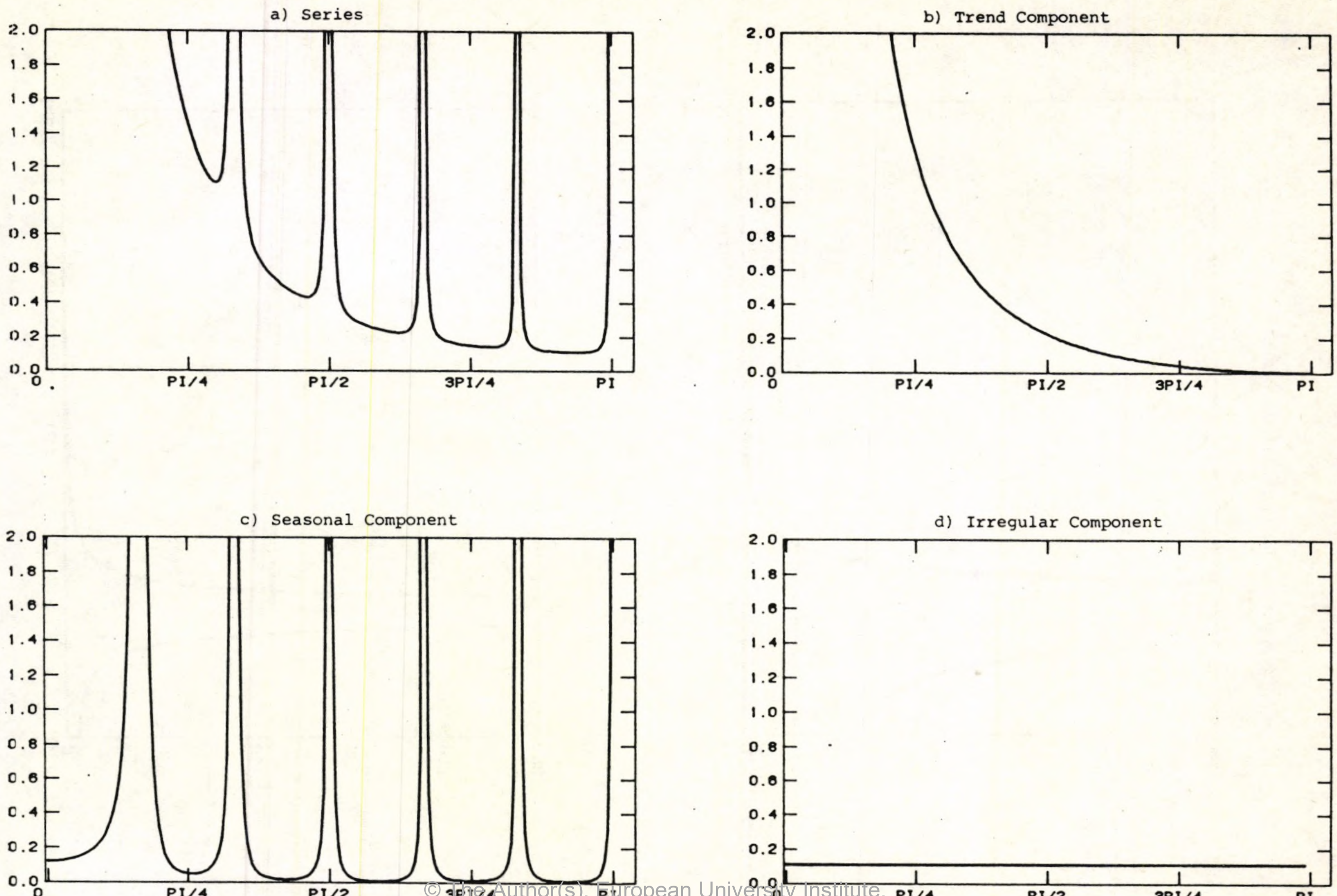
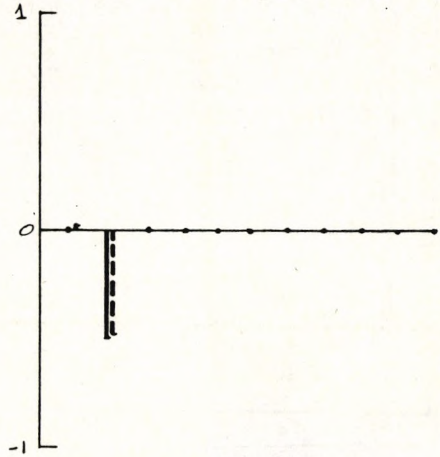


Figure 7

ACF of Theoretical Components
for Different Parameter Values

a) Trend Component



b) Seasonal Component

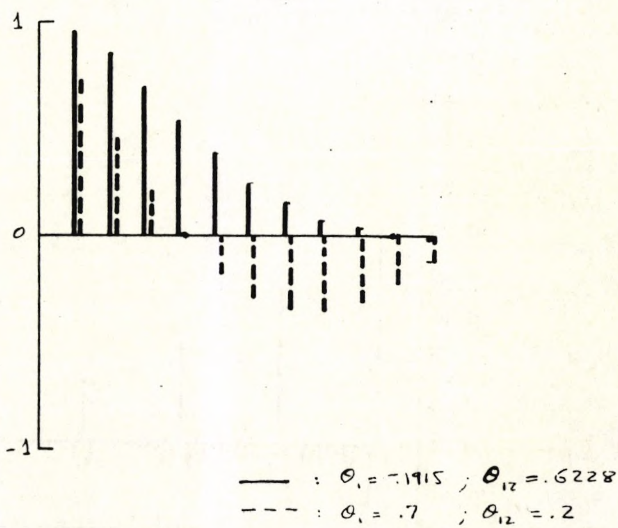
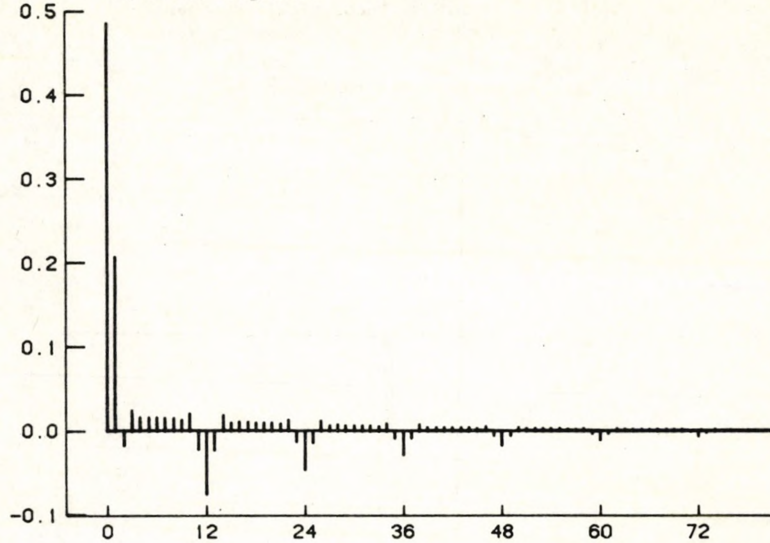


Figure 8

Filters for Estimators

a) Trend Component



b) Seasonal Component

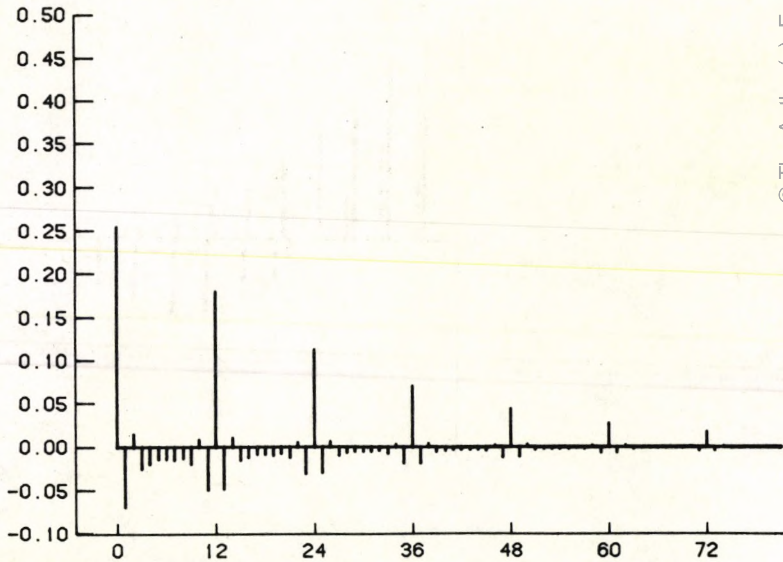


Figure 9

Estimated Components

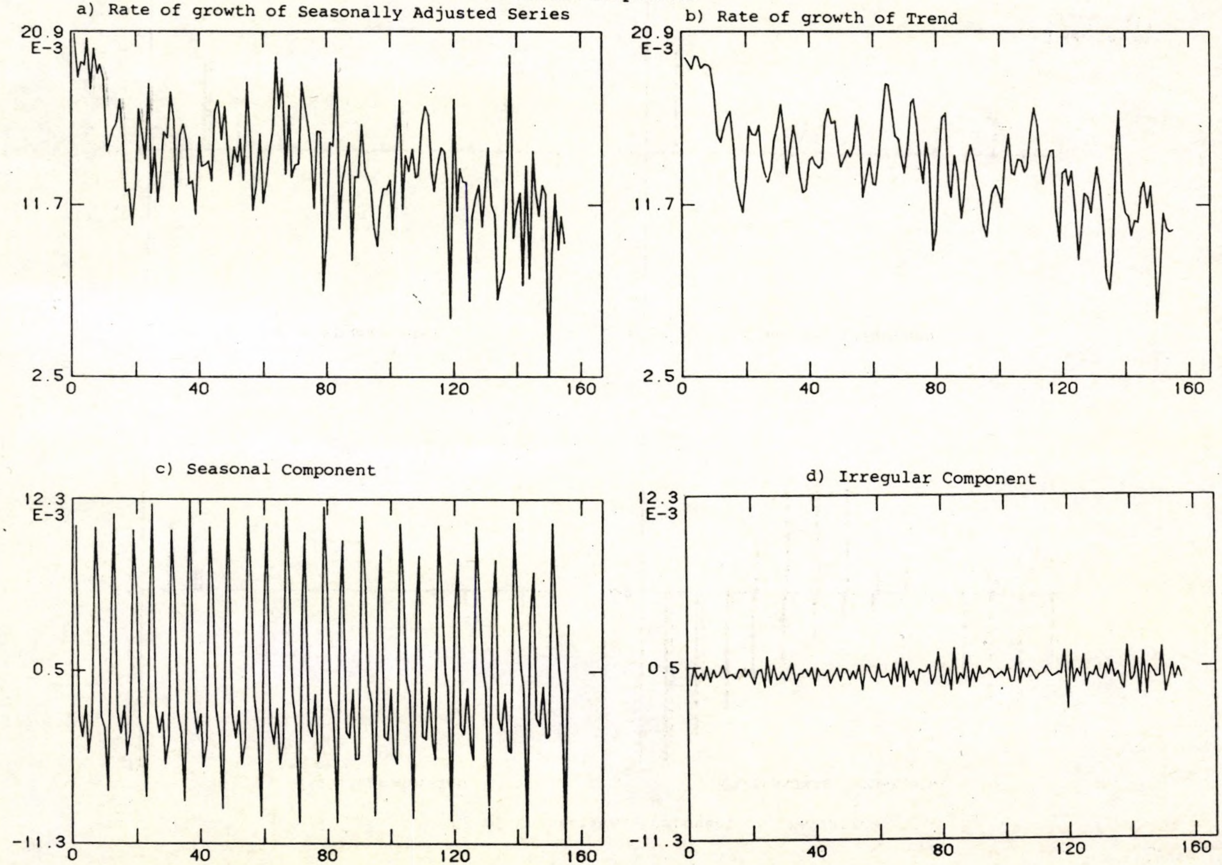


Figure 9

Figure 10

ACF of Theoretical Component and Estimator

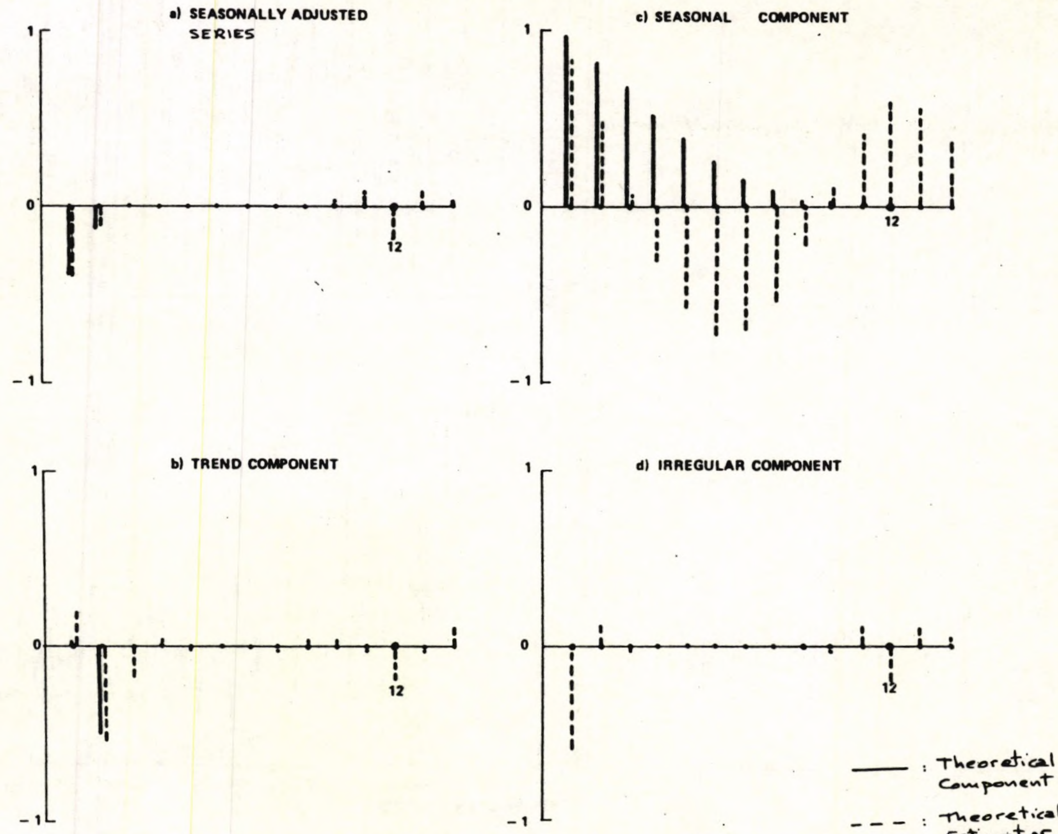
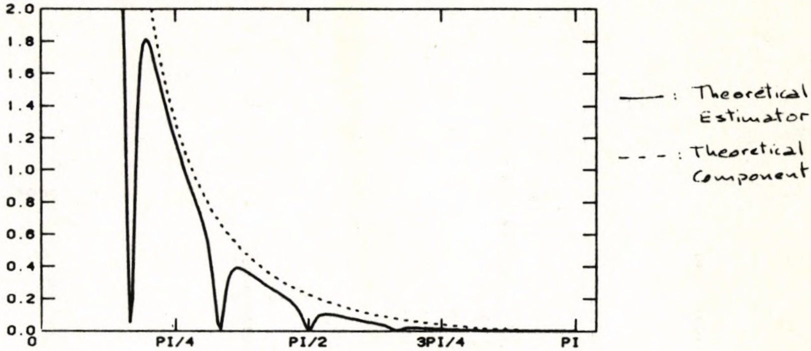


Figure 11

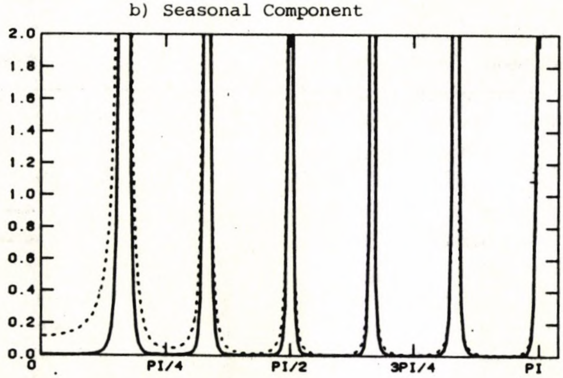
Spectra of Theoretical Components

and Estimators

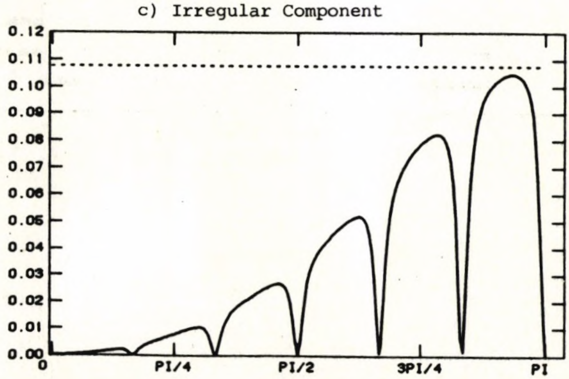
a) Trend Component



b) Seasonal Component

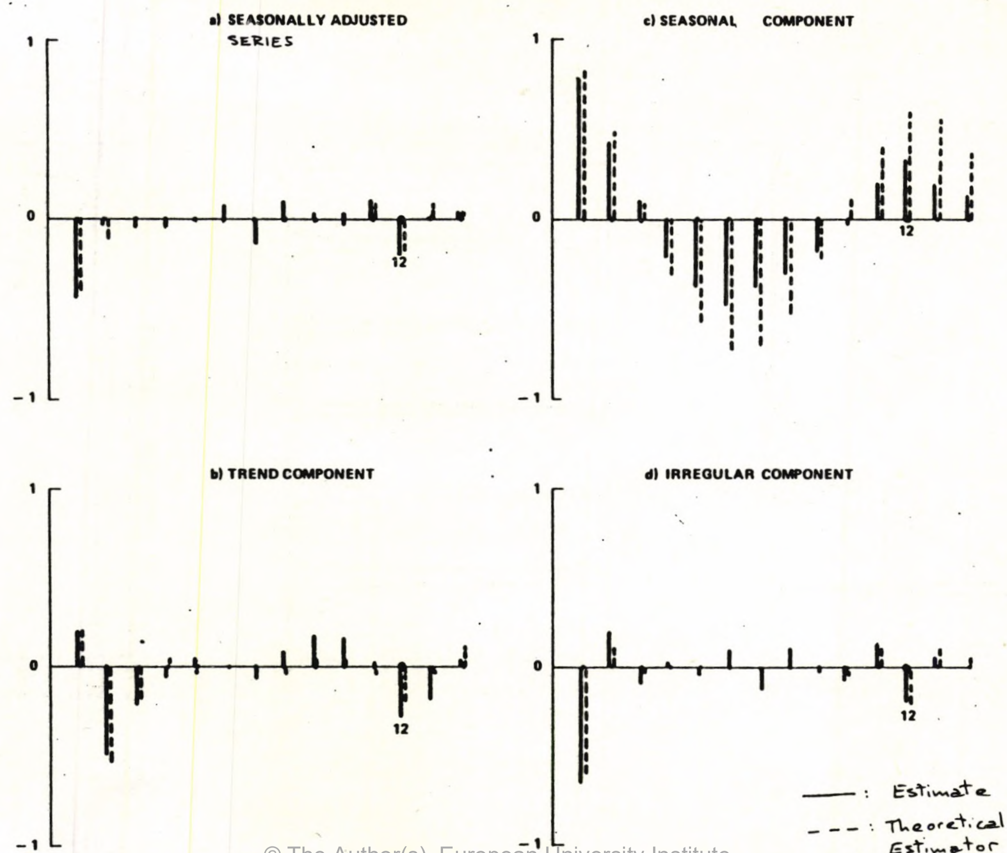


c) Irregular Component



ACF of Theoretical Estimator and Estimated Components

Figure 12



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